Coupled fluctuations of bulk plasma and the sheath in plasmas with $\mathbf{E}_0 \times \mathbf{B}_0$ drift

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Ion acoustic waves in plasmas with $\mathbf{E}_0\times\mathbf{B}_0$ electron drift become unstable due to the closure of plasma current in the chamber wall. Such unstable modes may enhance near-wall conductivity and bulk turbulent electron transport in plasma devices with $\mathbf{E}_0\times\mathbf{B}_0$ electron drift and unmagnetized ions. It is shown that the instability is sensitive to the wall material. The conditions and characteristics of this instability are analyzed for typical parameters of the Hall thrusters.

I. Introduction

It has been noted long ago that collisional transport of magnetically confined plasma is strongly affected by the closure of the parallel (along the magnetic field) electron current in the chamber walls. This is the so called Simon short circuit effect.[1–3] In recent work[4] it was shown that plasmas with $\mathbf{E}_0 \times \mathbf{B}_0$ electron drift and unmagnetized ions can be destabilized by the electron current admitted into the wall. The wall current, self-consistently determined from sheath boundary conditions, provides the positive feedback mechanism that renders the instability.

For typical parameters, classical collisional transport is not large enough to account for the plasma current and heating in devices with crossed $\mathbf{E}_0 \times \mathbf{B}_0$ that has been observed experimentally. Plasma instabilities are thought to be one of the main factor responsible for this anomalous cross field transport,[6–8] but the exact nature of these fluctuations remains unknown and is the subject of active research.[9–15] There is also significant experimental evidence that wall effects can also play a significant role in the cross field transport. The near wall conductivity caused by electron collisions with the channel walls and first proposed by Morozov,[16] is very sensitive to the sheath structure. The instability studied in Ref.[4] involve fluctuations of bulk plasma as well as the sheath, and thus may contribute to the near wall conductivity and turbulent transport of bulk plasma.

In this paper we further investigate the instabilities produced by the coupling of the $\mathbf{E}_0 \times \mathbf{B}_0$ driven sound waves in bulk plasma with sheath fluctuations[4] in some practical Hall thruster configurations.

II. Sheath induced modes

The sheath induced instabilities can be thought of as being the result of a positive feedback mechanism between the Doppler-shifted parallel (in the direction of the magnetic field $\mathbf{B}_0$) current and the fluctuations in plasma density and potential. The mechanism of such instability has been described in Ref.[4] Here we summarize the result and apply it to the channel region of two thruster configurations, namely the Stanford SPT-100 Hall thruster and the Technion's CAMILA thruster.

Consider a plasma between two symmetric material walls, such that the equilibrium magnetic field $\mathbf{B}_0 = B_0\widehat{\mathbf{z}}$ is normal to the wall, the equilibrium electric field is in the axial x-direction, $\mathbf{E}_0 = E_0\widehat{\mathbf{x}}$, so that the electrons drift along

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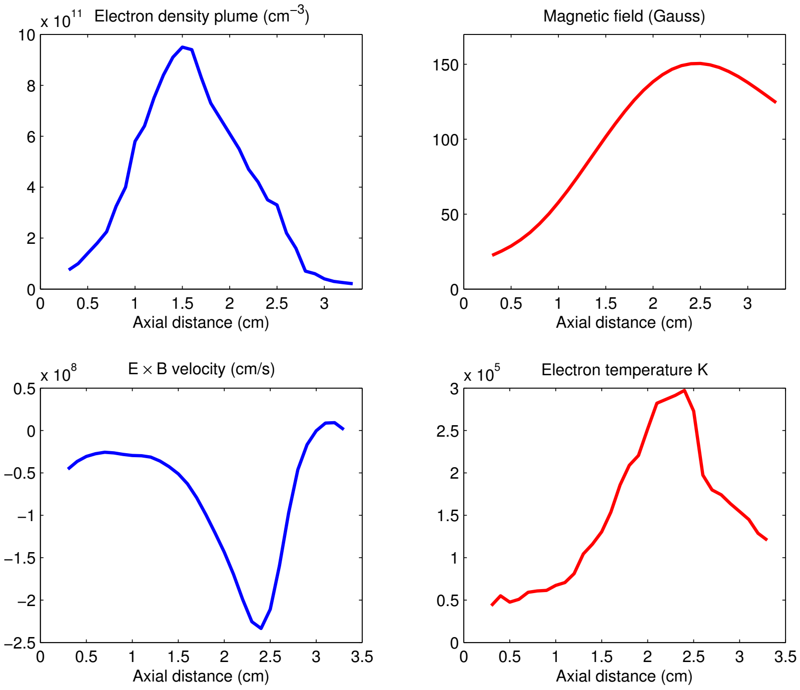


Figure 1: Plasma density, magnetic field, electron equilibrium drift velocity, u_0, and electron temperature profiles in SPT-100 Hall thruster obtained from HPHall-2 simulations as shown in Fig. 10 from Ref. 5. The exit plane is at x=2.5 cm.

the azimuthal, parallel to the walls, direction y, which is assumed to be periodic. For global, long wavelength modes, that are characterized by $H\partial/\partial z \ll 1$, where H is the channel width, the dispersion relation is given by[4]

$$\omega^2 \left(\omega + i\nu_{sh}\right) = \frac{\left|k_y\right| c_s \omega_{pi}^2}{\nu_{sh}} \left(\omega - \omega_0 + i\nu_{sh}\right), \tag{1}$$

where $\nu_{sh} = c_s/2H$ is the sheath collisional frequency. This dispersion relation has an unstable root for $\omega < \omega_0$ that corresponds to the reactive instability of the negative energy mode,[18] $\gamma \simeq \omega_r \simeq \left(-\omega_0 \left|k_y\right| c_s \omega_{pi}^2 / \left(\varepsilon \nu_{sh}\right)\right)^{\frac{1}{3}}$.

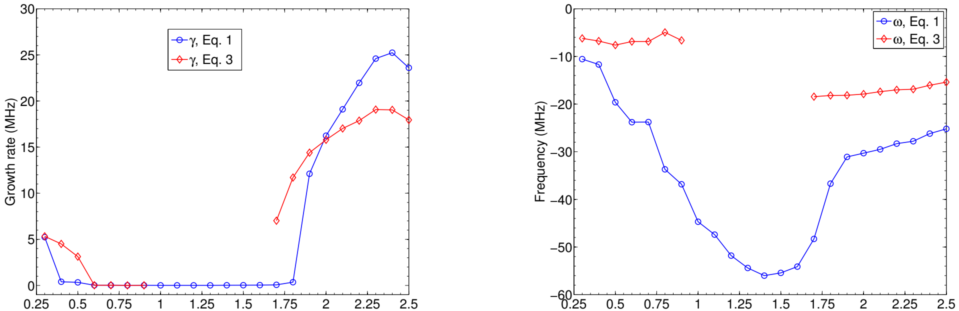
In the most general case, the sheath modes depend on both parallel (z) and perpendicular (y) coordinates. The eigen mode problem result in two coupled equations which have to solved simultaneously:

$$\omega^2 = (k_y^2 + k_z^2) c_s^2, \tag{2}$$

and

$$\begin{aligned}
&\frac{\tan\left(k_z H\right)}{k_z H} \times \left[\omega_0 - \frac{k_y^2 c_s^2}{\omega} - \frac{i\omega\left(\omega - \omega_0\right)}{\nu_{sh}} \left(1 - \frac{k_y^2 c_s^2}{\omega^2}\right)\right. \\
&\left. + i\omega \left(\frac{\omega^2}{k_y^2 c_s^2} - 1\right) \frac{K}{1 - iK}\right] \\
&+ \left[\omega - \omega_0 + \frac{\nu_{sh}}{1 - iK} - \nu_{sh} \frac{\omega^2}{k_y^2 c_s^2} \frac{K}{1 - iK}\right] = 0.
\end{aligned} \tag{3}$$

Solution of these equations determine the eigenvalues for ω. In the long wavelength limit, $k_z H \ll 1$, Eq. (1) can be recovered from Eq.(3).



(a) (b)

Figure 2: Growth rate and frequency of the instabilities in a SPT-100 thruster[5] as a function of axial distance to the anode as predicted by Eqs. (1) and (3). The exit plane is at x=2.5 cm.

III. Sheath induced instabilities in the SPT-100 thruster

Plasma parameters in the discharge chamber and near plume region the of SPT-100 Hall thruster were obtained by Hoffer with the HPHall-2 code[19, 20] as reported in Ref.5. These plasma profiles are in good agreement with available experimental data for the SPT-100 thruster and can be seen in Fig. 1. The SPT-100 thruster has dielectric walls made of Borosil ($BNSiO_2$) with a dielectric constant, ε, of 3.50-3.75. The channel width is 0.75 cm.[5] The growth rate and frequency obtained from Eqs. (1) and (3) are shown in Fig. 2. The instability is present mainly in the near anode region and the exit plane. In the mid channel region, the global modes from Eq. (1) predict a small, albeit not zero, growth rate (of the order of 10 KHz), while the small scales modes from Eq. (3) predict stability in the mid channel region. The largest growth rates are attained close to the exit plane, reaching a value of 25 MHz for the global modes and 19 MHz for the local modes, both at x = 2.4 cm. Both the global and local modes exhibit a negative real part of the frequency, suggesting that they propagate in the direction of the $\mathbf{E}_0 \times \mathbf{B}_0$ equilibrium flow. In the mid channel region where the global modes have growth rates of the order of 10 KHz, the frequency reaches -56 MHz. Outside of this region, the maximum value of the real part of the frequency is 31 MHz for the global mode and of 18 MHz for the local modes.

IV. Sheath induced instabilities in the CAMILA thruster

The coaxial magnetoisolated longitudinal anode thruster (CAMILA) was developed at the Technion's Asher Space Research Institute is an effort to adapt Hall thruster technology to low power regimes.[17] This thruster is characterized by having a longitudinal magnetic field inside the anode cavity that reduces the electron mobility in the radial direction. A radial electric field is created in the direction towards the center of the channel. Two configurations are currently under development, simplified CAMILA, without anode coils and full CAMILA, with anode coils. In the following we will refer to the simplified version of the thruster. A more detailed description of the CAMILA concept can be found in Ref. 17 and references therein. The plasma parameter profiles for the CAMILA thruster are shown in Fig. 3.[17] The growth rate and frequencies of the unstable modes calculated from Eqs. (1) and (3) are shown in Fig. 4.

Similar to the SPT-100 thruster, the instability is also present mainly in the near anode region and close to the exit plane, there is also a mid channel region where the global modes exhibit a small growth rate. The local mode equation predicts stability in this mid channel region. The maximum growth rate is 20 MHz for the global modes and 15 MHz for the local modes. Both these maxima are reached close to the exit plane. Both the global and local modes exhibit a negative real part of the frequency, suggesting that they propagate in the direction of the $\mathbf{E}_0 \times \mathbf{B}_0$ equilibrium flow. In the mid channel region where the global modes have growth rates of the order of 10 KHz, the frequency reaches an absolute value of 71 MHz. In the near exit region, the global modes reach an absolute value of the frequency of 31 MHz, while the local modes have a maximum absolute value of the frequency of 23 MHz.

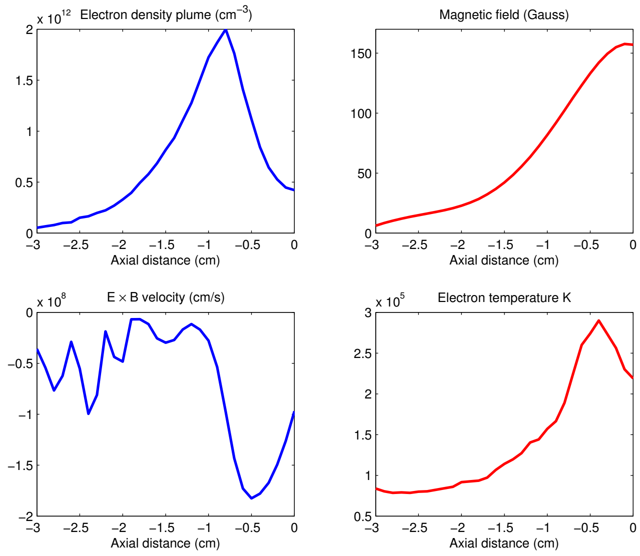
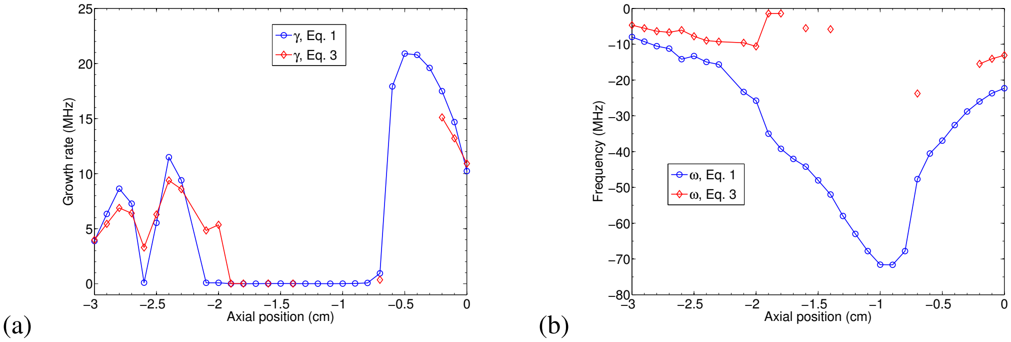


Figure 3: Plasma density, magnetic field, electron equilibrium drift velocity, u_0, and electron temperature profiles in CAMILA Hall thruster from Ref. 17. The exit plane is at x=0.



(a) (b)

Figure 4: Growth rate and frequency of the instabilities in the CAMILA thruster[17] as a function of axial distance to the anode as predicted by Eqs. (1) and (3). The exit plane is at x=0.

V. Conclusions

The feedback mechanism between the parallel current to the sheath and the density and potential fluctuations in a plasma give rise to an instability involving bulk plasma and sheath fluctuations. In this work, we have applied the results obtained in a recent work[4] to some Hall thruster parameters under the smallest azimuthal wavenumber. The resulting instability is mainly confined to the near anode and exit plane regions. The global modes have a small growth rate and high value of the real part of the frequency in the mid channel region. The local modes predict stability in this mid channel region.

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