Implementation of an Entropy Closure Model for Two-dimensional Hybrid Hall Thruster Simulations

IEPC-2013-216

Presented at the 33^{rd} International Electric Propulsion Conference,
The George Washington University, Washington, D.C., USA
October 6–10, 2013

Eunsun Cha* and Mark A. Cappelli†
Stanford University, Stanford, CA, 94305, USA

and

Eduardo Fernandez‡
Eckerd College, St Petersburg, FL, 33711, USA

Abstract: An entropy closure formulation for Hall thrusters has been implemented in a radial-axial hybrid simulation to validate the reliability and transportability of the model. We modeled the entropy production and its scaling with effective collision frequency and magnetic field, and in a transport equation for electron entropy, which led to a first-order ordinary differential equation for electron mobility. With the implementation of this differential equation, electron mobility is simultaneously computed in the simulation and used in the computation of electron temperature and the resulting plasma properties. This paper presents the simulation results with the entropy model on the Stanford Hall thruster geometry and compares these to the results of existing hybrid simulations with experimental mobility and Bohm models. The discharge voltage is varied to further validate the model. The entropy closure model is found to perform as well as simulations carried out using the experimentally measured mobility.

Nomenclature

B	= magnetic field strength
ν_{eff}	= effective collision frequency
n_e	= electron number density
T_e	= electron temperature
$\hat{n}$	= direction perpendicular to the magnetic field
$\mu_{e\,\text{eff}}$	= effective electron mobility
$\dot{n_e}$	= volumetric ionization rate
s_e	= electron entropy
$\dot{s_e}$	= volumetric entropy production rate
ω_{ce}	= electron cyclotron frequency
$\vec{u}$	= velocity
$\vec{q}$	= heat flux

*Research Assistant, Department of Mechanical Engineering, escha@stanford.edu.
†Professor, Department of Mechanical Engineering, cap@stanford.edu.
‡Professor, Department of Mathematics and Physics, fernane@eckerd.edu.

I. Introduction

The Hall-effect thruster (HET) is a state-of-art technology for satellite station-keeping, orbit maintenance, and orbit transfer. It has been widely studied by many researchers who have modified its traditional design for concepts that could make it suitable for various applications. Those applications include their use as high-power thrusters for a primary in-space propulsion system[1], air-breathing thrusters for low earth orbit missions, and hybrid engines that utilize the propellants that are shared with other on-board propulsion systems. The cost of xenon, which has increased significantly in recent years, has led to the development of HETs that operate on alternative propellants. Many researchers have expanded the propellant base to include iodine[2,3], bismuth, krypton[4], nitrogen[5], and even air[6,7].

Performance optimization of Hall thrusters and even design for alternative propellants will benefit from reliable computational tools because ground-based experiments and life testing are costly. In the Stanford Plasma Physics Laboratory (SPPL), we have been developing a two-dimensional hybrid particle-in-cell (PIC) simulation[8]. An important but poorly represented parameter in these simulations is the cross-field electron mobility, because establishing a reliable and stable model for the electron transport has been one of the greatest challenges in Hall thruster research. Measurements[9] of electron cross-field transport seem to imply that it is anomalously high – much higher than that described by classical collisions. Although models like those described by Bohm-type scaling with the magnetic field[13] are relatively straightforward to implement, they do not seem to capture the spatial variations in plasma properties throughout the Hall thruster channel. Early theories suggested the possible role played by electron-wall scattering in enhancing the so-called near-wall conductivity, although it appears that the loss of the high energy electrons to the wall may not be replenished in the bulk plasma at a sufficient rate to account for the anomalous electron current. Researchers have used, with some success, ad-hoc types of models with arbitrary coefficients or combinations of the early theories. However, the ad-hoc models seem to be incapable of capturing plasma properties, and often depend on some adjustable parameters derived from experiments. Thus, developing a suitable physical model for electron transport is one of the highest priorities for robust simulations of Hall thrusters because this would allow engineers to better explore their design space.

Recently, Knoll et al. developed a one-dimensional simulation that incorporated an isentropic description for the electron fluid[10]. However, its implementation[11] into a two-dimensional simulation has been difficult for two reasons. First, the isentropic model has an inherently positive feed-back characteristic, which hampers the convergence. Second, early results of simulations for a limited set of thruster conditions suggested the possible failure of the isentropic assumption in some regions of the thruster channel. In essence, the assumption of zero entropy production may not be valid in the regions where electron-scattering collisions are very frequent.

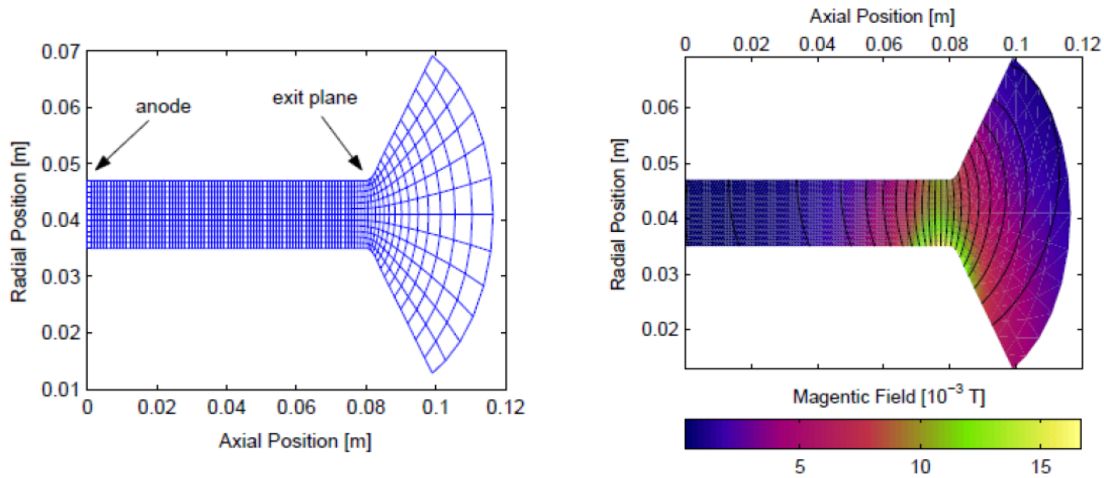
Recently, we have introduced a new method, which we call as an entropy closure model, to describe the electron transport in Hall thrusters. In this approach, we model the entropy production and its scaling with effective collision frequency, ν_{eff}, and magnetic field, B, and use a transport equation for electron entropy to close the set of equations for the electron fluid. The transport equation for electron entropy allows us to calculate the electron drift velocity, which defines the electron mobility in accordance with the electron momentum equation used in most Hall thruster simulations. Electron mobility therefore becomes a calculated parameter within the framework of a simulation.

In this study presented here, we focused on the practical implementation of the entropy closure model, which is described in more detail in a companion paper[12]. The initial implementation of the model used as a testbed laboratory Hall thruster (Stanford Hall thruster, SHT) operating at 200 V. The results of the simulation were compared to the experimental measurements and to the simulations carried out with the other transport models. To examine the transportability of the entropy closure model, simulations were carried out over a range of discharge voltages.

II. Numerical Model

A. Hall Thruster Simulation

The Hall thruster simulation used in this study is built on the same simulation platform as that widely used by others in the SPPL. This initial simulation was developed by Fernandez et al.[8], and is similar to that of Fife at MIT[14]. The simulation uses the radial-axial plane of the Hall thruster as computational domain. As depicted in Fig. 1a, the domain has an 8-centimeters-long channel that spans 1.2 centimeters in



(a) (b)

Figure 1: (a) Radial-axial domain of the Stanford Hall thruster. (b) Magnetic field

the radial direction. The imposed magnetic field, shown in Fig. 1b, was based on measurements made on the laboratory thruster.

The simulation is composed of a Particle-In-Cell (PIC) components that pushes the heavy plasma constituents (ions and neutrals) and a fluid model for the lighter and magnetized electrons. These two components are coupled by quasi-neutrality ($n_e \approx n_i$). In the PIC component, each ion superparticle represents about 10^6 to 10^8 actual particles, while each neutral superparticle represents about 10^8 to 10^{10} actual particles. The computational domain is two-dimensional in the radial-axial plane, and we assume that the plasma properties are axi-symmetric around the annulus of the Hall thruster. Despite this assumption of axi-symmetry, the motion of the ions and neutrals are tracked in three-dimension, whereas the electrons are treated as a quasi 1-dimensional conducting fluid.

Neutral superparticles are injected from the center of the anode based on the prescribed mass flow rate and are ionized by electron-impact to produce singly-ionized atoms/molecules. The ionization process is modeled as a function of electron temperature. This ion production serves as a source in the electron continuity equation as well as an energy drain in the electron energy equation. The energy lost through ionization includes cost-factors to indirectly account for non-ionizing collisions which ultimately result in internal energy and radiation losses. The heavy particles are advanced in time through solution of their respective equations of motion in cylindrical coordinates. The background gas effect is also examined for the SHT simulation by applying a finite background gas pressure, 0.05 mTorr. Although previous research by our group examined the role played by charge exchange collisions[15], this effect is not included in the results presented in this study. Further details about the simulation algorithm and the numerical scheme can be found in Ref. 16.

B. Entropy Closure Model

The entropy closure model is based on the assumption that in the regions of strong magnetic field, effective collisions are less frequent, thus the rate of entropy production is expected to be small[12]. This assumption guides the selection of a maximal set of parameters to determine the dimensionless relation that governs the local-scaled entropy production, $\dot{s_e}/k_B$. The set includes the plasma density, n_e, the effective collision frequency, ν_{eff}, the applied magnetic field, and B (through $\omega_{ce} = eB/m$). The scaling analysis leads us to the following relation:

$$\dot{s}_e = n_e k_B \nu_{\text{eff}} \cdot f\left(\frac{\omega_{ce}}{\nu_{\text{eff}}}\right). \tag{1}$$

The functional dependence of f must be determined by empirical means, but we will test the simplest of functions – one that is linear in the inverse Hall parameter, i.e.,

$$f\left(\frac{\omega_{ce}}{\nu_{\text{eff}}}\right) \approx \alpha\frac{\nu_{\text{eff}}}{\omega_{ce}}, \tag{2}$$

where α is constant. In this study, we take α to be of order of unity.

A transport equation for entropy, s_e, is used to close the set of equations for the electron fluid, where the existing set of equations are the first three moments of the Boltzmann equation.

$$\frac{\partial(n_e s_e)}{\partial t} + \nabla \cdot n_e s_e \vec{u_e} = -\nabla \cdot \frac{\vec{q_e}}{T_e} + \dot{s_e}. \tag{3}$$

Together with Eqs. (1) and (2), this entropy transport equation can be shown[12] to reduce to the following first-order ordinary differential equation for a 1-D simulation in which properties are assumed to vary mainly along the direction normal to the magnetic contour, i.e., along the $\hat{n}$ direction.

$$\frac{8}{\pi}\frac{k_B T_e}{e}\frac{\partial \ln T_e}{\partial \hat{n}}\frac{d\mu_{e\text{eff}}}{d\hat{n}} + \frac{eB^3}{m_e}\mu_{e\text{eff}}^2 + \beta\,\mu_{e\text{eff}} - \frac{\dot{n_e}s_e}{k_B n_e} = 0. \tag{4}$$

Here, T_e is electron temperature, and $\dot{n_e}$ is the ionization rate. The variable β in Eq. (4) is

$$\begin{aligned}\beta = \frac{k_B T_e}{e}\left[\frac{eE}{k_B T_e} + \frac{d\ln n_e}{d\hat{n}} + \frac{d\ln T_e}{d\hat{n}}\right]\left[\frac{3}{2}\frac{d\ln T_e}{d\hat{n}} - \frac{d\ln n_e}{d\hat{n}}\right] \\ + \frac{8}{\pi}\frac{k_B T_e}{e}\left[\frac{d\ln T_e}{d\hat{n}}\frac{d\ln n_e}{d\hat{n}} + \left(\left(\frac{d\ln T_e}{d\hat{n}}\right)^2 + \frac{d^2\ln T_e}{d\hat{n}^2}\right)\right].\end{aligned} \tag{5}$$

C. Implementation Details

There are several possible approaches in the numerical application of the Eq. (4). In this study, we chose to solve a quadratic equation for $\mu_{e\text{eff}}$ by setting the first order derivative, $\frac{d\mu_{e\text{eff}}}{d\hat{n}}$, as a constant at the given timestep and determined by the previous timestep. With this numerical strategy, we benefit in two ways. First, the computation cost is significantly reduced in this way, because no iteration is necessary for each node at each timestep. Second, and more importantly, we avoid the challenging singularity problem that is caused by the structure of the coefficient to the first order derivative of $\mu_{e\text{eff}}$. The analysis of Ref. 12 shows that the contribution to the resulting mobility from the 1st order derivative term is small, justifying this approach.

We solve the Eq. (4) along the centerline of the radial dimension and use it as a representative value for each axial position for the purposes of both numerical stability and cost efficiency. The derivatives of the logarithms of electron temperature and density($\frac{d\ln T_e}{d\hat{n}}$, $\frac{d\ln n_e}{d\hat{n}}$) are calculated using a second order Newton's method at each timestep, with the computed data from the previous timestep.

Numerical adjustments were adopted after a series of preliminary studies with the entropy closure model in order to enhance the stability. These include attention to the region close to the anode when using the effective mobility as calculated by Eq. (4). In our current version of this hybrid simulation, the computational grid near the anode is not well resolved due to the small variations in the magnetic field in this region. This led to some numerical instability which was avoided by enforcing the classical mobility in that region. In the rest of the domain, the computed mobility is introduced gradually at each time step, with 10% of the instantaneously computed mobility at each timestep added to 90% of the mobility used in the previous timestep. The instantaneously computed mobility is also filtered by local averaging to reduce high frequency spatial noise.

III. Results and Discussion

A. Comparison of Transport Models

Figure 2 is a representative example of the entropy closure model performance in the hybrid simulation for the SHT. The time-averaged profiles of the computed inverse Hall parameter and the effective mobility are compared to the experimentally measured data, a Bohm model, and the transport computed based on

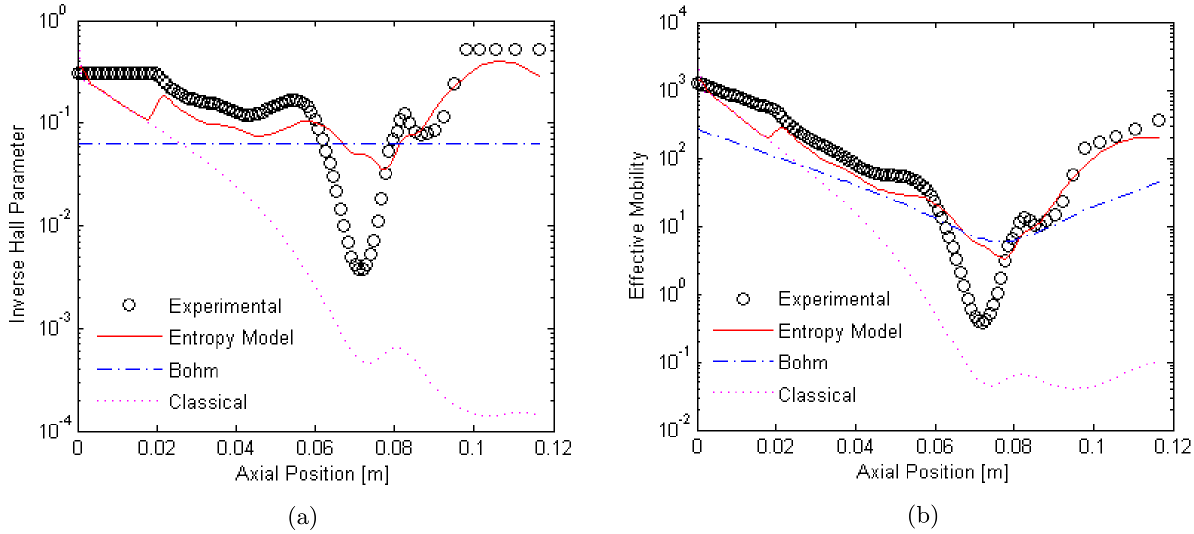


Figure 2: (a) Comparison of experimentally measured inverse Hall parameter with transport models. (b) Comparison of experimentally measured electron mobility. Discharge voltage is 200 V.

	Discharge Current [A]
Experimental Measurement	2.7
Simulation with Entropy Model	1.96
Simulation with Entropy Model and Background Gas	2.41
Simulation with Experimental Mobility	1.88
Simulation with Bohm Mobility	1.75

Table 1: Comparison of simulated and experimentally measured discharge current. Discharge voltage is 200 V.

classical scattering. The nominal operating condition for the SHT is used (discharge voltage of 200 V and the xenon mass flow rate of 2 mg/s). The calculated electron transport using the entropy closure model captures the transport barrier, although weak, near the exit plane (where the axial position is 0.08 m) and agrees reasonably well with that measured at the axial positions between 0.02 to 0.04 m and outside the channel where the axial position is greater than 0.08 m.

Table 1 compares the simulated discharge currents to the experimental measurement. The discharge current computed using the entropy closure model is 1.96 A. This value is close to that computed using the Bohm and classical models. However, when the effect of background gas is included, with the same pressure as experimental condition (0.05 mTorr), the computed discharge current is raised up to 2.41 A. It is noteworthy that the discharge current is strongly influenced by the background pressure and also that without any fitting procedure, the entropy model predicted the current close to the measured current, although with about 11% underestimation.

Figure 3 shows the comparison of the axial profiles of the simulated potential, axial ion velocity, electron temperature, and plasma density to those measured experimentally. The entropy closure model predicts the potential drop in the acceleration region to be more gradual than that seen in the experimental measurements. This is consistent with the weaker transport barrier generated using the entropy closure model. The entropy model prediction for the axial ion velocity, however, is in remarkably good agreement with the measurements. This is encouraging, considering that the ion velocity measurements carry some of the smallest experimental uncertainty. Note that the experimental ion velocity data (○) are the most probable values from laser induced fluorescent velocimetry[17], while the simulated data (lines) are mean values of all ion particles. Therefore, the lower ion velocity (outside the channel) of the simulated data with entropy model and the background

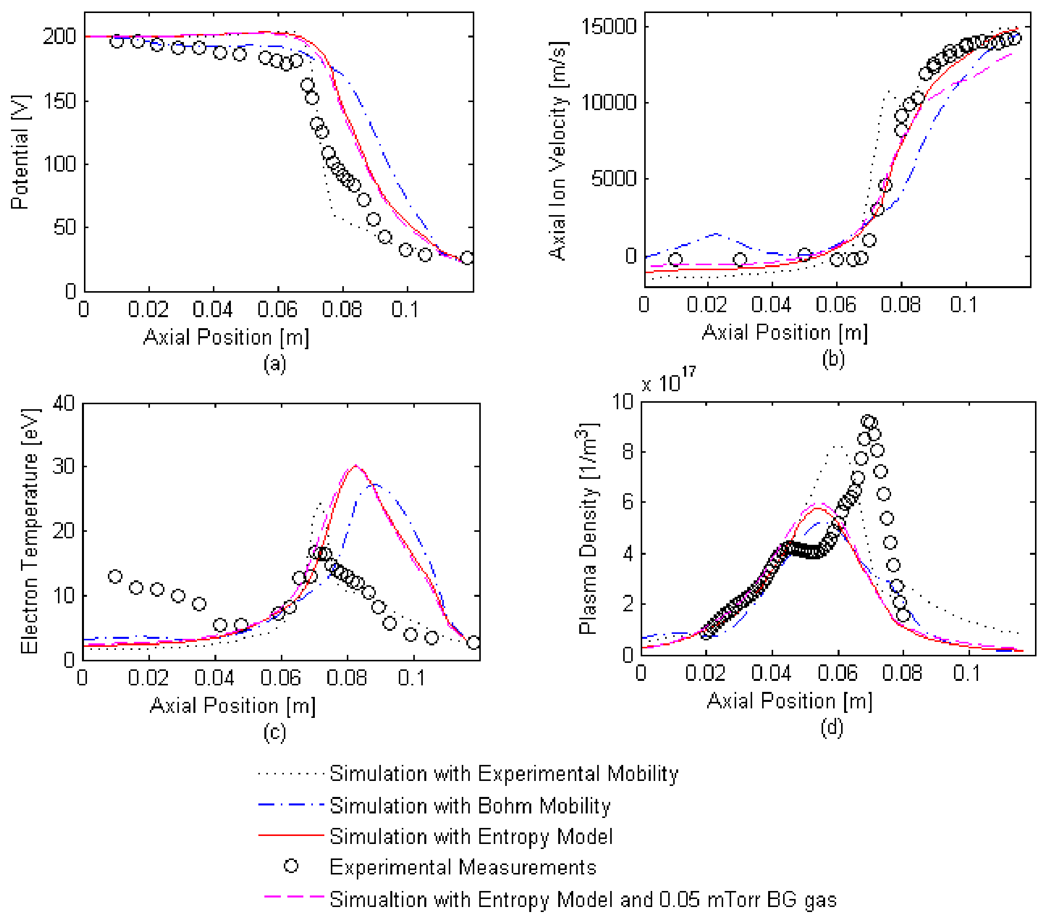


Figure 3: Comparison of experimental measurements of plasma properties with simulated results using Bohm, experimental, and entropy model mobility. Discharge voltage is 200 V.

gas effect (dashed line) is meaningful because it accounts for the increased number of ions with lower speeds outside the channel due to the background pressure. The model-simulated electron temperature is seen to be approximately 50% higher at the peak than those measured experimentally. However, the location of this computed peak is closer to the peak location measured measured experimentally than is the peak location computed using the Bohm model.

B. Discharge Voltage Variation

Discharge voltages other than the nominal 200 V were tested to study the portability of the entropy closure model. In Fig. 4, 160 V and 100 V cases are compared to the experimental measurements as well as the simulated results that were obtained using the experimental mobility. The computed mobility for 100 V, 160 V, and 200 V are compared in Fig. 5. The mobility calculated with the entropy closure model is not very sensitive to the discharge voltage, although the calculated plasma properties seem to vary, albeit weakly, with the change in discharge voltage. This is expected from the nature of the mobility equation because the effective mobility depends on the logarithms of plasma density and electron temperature.

Figure 6 compares the I-V characteristics computed by the entropy closure model to that measured experimentally, as well as that computed using the experimentally measured mobility. The I-V characteristics predicted by the entropy closure model show a trend similar to that in the experiments. It is noteworthy that this trend is not seen when the measured mobility is used in the simulations, particularly for the low discharge voltage cases. This suggests that the mobility is not as sensitive to operating conditions as suggested in the early experiments of Ref. 9.

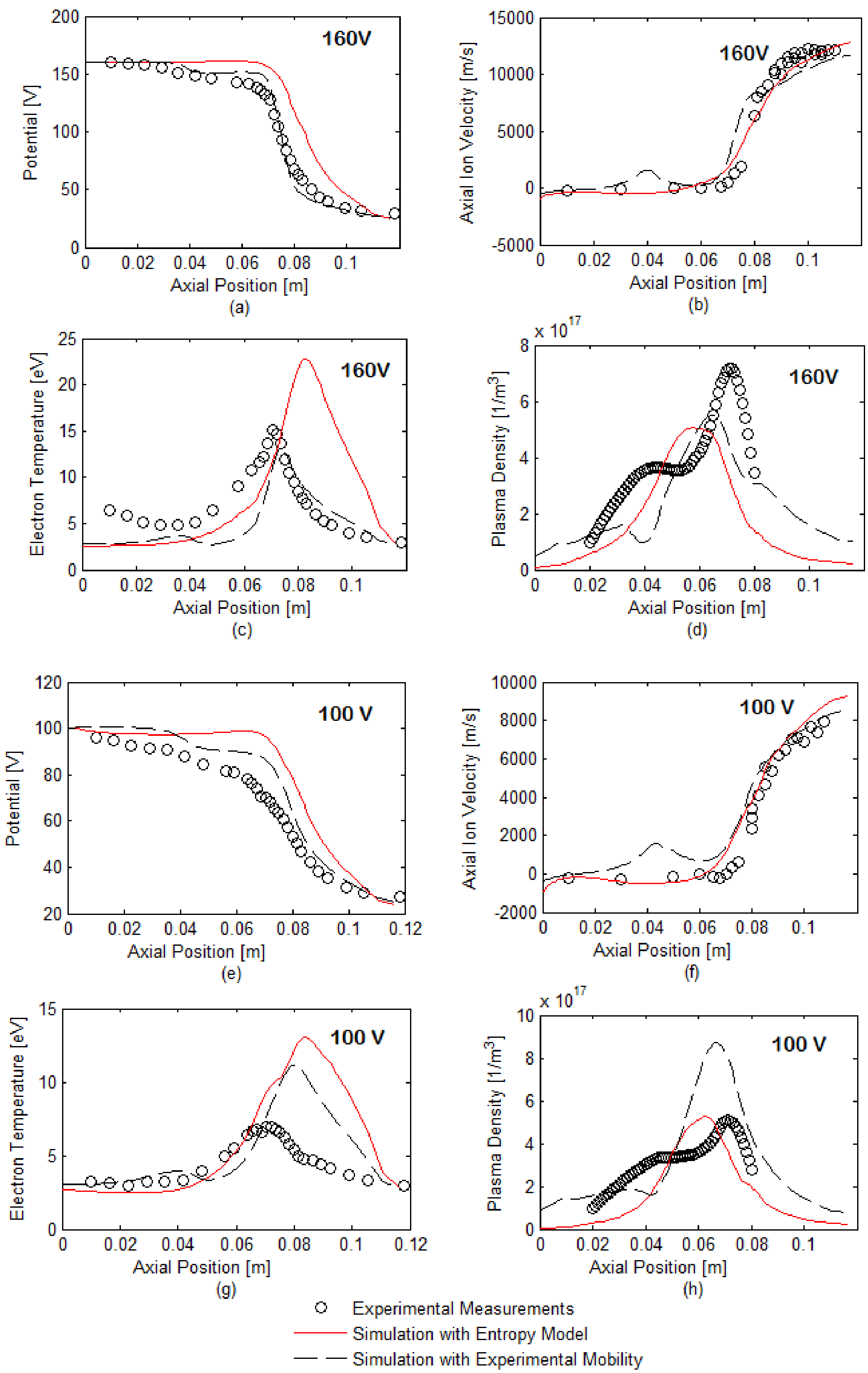


Figure 4: Comparison of experimental measurements of plasma properties with simulated results using experimental mobility, and entropy model. (a)-(d) Discharge voltage is 160 V. (e)-(h) Discharge Voltage is 100 V.

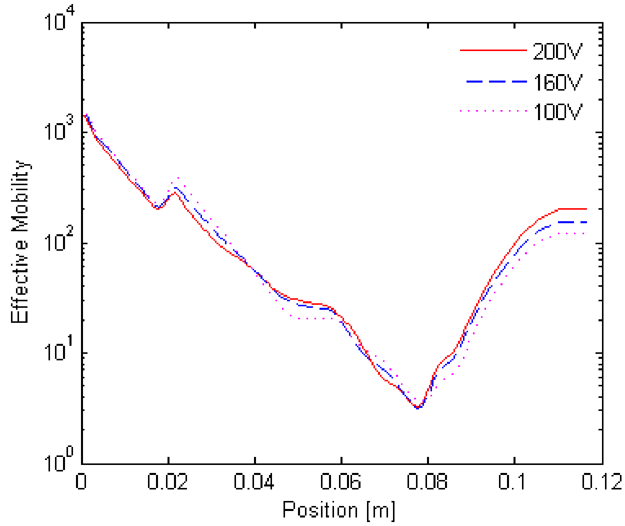


Figure 5: Comparison of simulated mobility using entropy model at various discharge voltages.

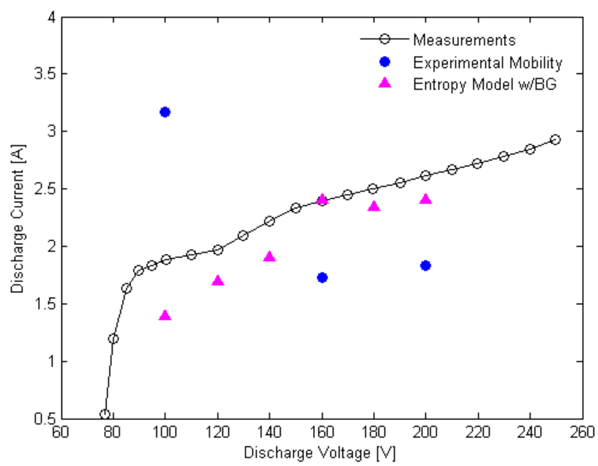


Figure 6: Relations between discharge voltage and current are compared for the SHT. Xenon mass flow of 2 mg/s are used for all data.

IV. Summary

This paper describes the introduction of an entropy closure model into hybrid simulations of Hall thrusters. The model uses dimensional scaling guided by experiments to prescribe the rate of entropy produced due to effective electron scattering and uses this rate in an entropy transport equation to close the set of equations for the electron fluid. The entropy transport equation removes the need to provide an independent specification of the electron mobility because it falls out of the simulations as a computed property. We compare the performance of this model to experimental measurements conducted on the Stanford Hall thruster. This entropy closure model is found to perform as well as simulations carried out using the experimentally measured mobility, and in some operating regions (e.g., low voltage) better captures the trends in discharge current seen experimentally. The model currently under-predicts the measured discharge current, and we attribute this to parameters related to the treatment of electron energy loss during volume ionization and electron-wall collisions. Future work will address this additional physics and will also include studies of how well this model predicts the performance of other xenon-fueled Hall thrusters, such as the

SPT-100, as well as thrusters operating on alternative propellants.

Acknowledgments

E. Cha thanks the United States National Science Foundation (NSF) for sponsoring the IEPC travel grants. M. A. Cappelli thanks the U.S. Air Force Office of Scientific Research and Dr. Mitat Birkan for support of this research.

References

[1] Koppel, C. R., and Estublier, D., "The SMART-1 Hall Effect Thruster Around the Moon: In Flight Experience," *Proceedings of the 29th International Electric Propulsion Conference*, The Electric Rocket Propulsion Society, Worthington, OH, 2005, pp. IEPC-2005-119.

[2] Hillier, A. C., "Revolutionizing Space Propulsion Through the Characterization of Iodine as Fuel for Hall-Effect Thrusters," Ph.D. Dissertation, The Department of Aeronautics and Astronautics, Air Force Institute of Technology, Air University, Air Education and Training Command, Wright-Patterson AFB, OH, 2011.

[3] http://www.busek.com/technologies__hall.htm

[4] Scharfe, D. B., "Alternative Hall Thruster Propellants Krypton and Bismuth: Simulated Performance and Characterization," Ph.D. Dissertation, The Department of Mechanical Engineering, Stanford University, Stanford, CA, 2009.

[5] Cha, E., Scharfe, D. B., Scharfe, M. K., Cappelli, M. A., and Fernandez, E., "Hybrid Simulations of Hall Thrusters Operating on Various Propellants," *Proceedings of the 31st International Electric Propulsion Conference*, The Electric Rocket Propulsion Society, Worthington, OH, 2009, pp. IEPC-2009-075.

[6] Cifali, G., Misuri, T., Rossetti, R., Andrenucci, M., Valentian, D., Feili, D., and Lotz, B., "Experimental Characterization of HET and RIT with Atmospheric Propellants," *Proceedings of the 32nd International Electric Propulsion Conference*, The Electric Rocket Propulsion Society, Worthington, OH, 2011, pp. IEPC-2011-224.

[7] Garrigues, L., "Study of a Hall Effect Thrusters Working with Ambient Atmospheric Gas as Propellant for Low Earth Orbit Missions," *Proceedings of the 32nd International Electric Propulsion Conference*, The Electric Rocket Propulsion Society, Worthington, OH, 2011, No. IEPC-2011-142.

[8] Fernandez, E., Cappelli, M. A., and Mahesh, K., "2D Simulations of Hall Thrusters," *CTR Annual Research Briefs*, 1998, pp. 81.

[9] Meezan, N. B., Hargus, Jr., W. A., and Cappelli, M. A., "Anomalous Electron Mobility in a Coaxial Hall Discharge Plasma," *Physical Review E*, Vol. 63, 2002, pp. 026410.

[10] Knoll, A. K., and Cappelli, M. A., "A Simple Isentropic Model of Electron Transport in Hall Thrusters," *Journal of Physics D: Applied Physics*, Vol. 41, No. 16, 2008.

[11] Cha, E., Cappelli, M. A., and Fernandez, E., "Implementation of a Polytropic Model for Two-dimensional Hybrid Hall Thruster Simulations," *Bulletin of the American Physics Society, 65th Annual Gaseous Electronics Conference*, Vol. 57, No. 8, 2012.

[12] Cappelli, M. A., Cha, E., and Fernandez, E., "Entropy Closure Model for Transport in Hall Thruster Simulations," *Proceedings of the 33rd International Electric Propulsion Conference*, The Electric Rocket Propulsion Society, Worthington, OH, 2013, pp. IEPC-2013-088.

[13] Bohm, D., Burhop, E., and Massey, H., *The Characteristics of Electrical Discharges in Magnetic Fields*, McGraw Hill, New York, NY, 1949, p. 13.

[14] Fife, J. M., "Nonlinear Hybrid-PIC Modeling and Electrostatic Probe Survey of Hall Thrusters," Ph.D. Dissertation, The Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA, 1998.

[15] Allis, M. K., Gascon, N., Cappelli, M. A., and Fernandez, E., "Effect of Charge Exchange on 2D Hall Thruster Simulation," *Proceedings of the 29th International Electric Propulsion Conference*, The Electric Rocket Propulsion Society, Worthington, OH, 2005, pp. IEPC-2005-057.

[16] Scharfe, M. K., "Electron Cross Field Transport Modeling in Radial-Axial Hybrid Hall Thruster Simulations," Ph.D. Dissertation, The Department of Mechanical Engineering, Stanford University, Stanford, CA, 2009.

[17] Hargus, Jr., W. A., "Investigation of the Plasma Acceleration Mechanism within a Coaxial Hall Thruster," Ph.D. Dissertation, The Department of Mechanical Engineering, Stanford University, Stanford, CA, 2001.