2D Axisymmetric Fluid and Electromagnetic Models for Inductively Coupled Plasma (ICP) in RF Ion Thrusters

IEPC-2013-294

Presented at the 33^{rd} International Electric Propulsion Conference, The George Washington University, Washington, D.C., USA October 6–10, 2013

Emre Turkoz* and Murat Celik†

Bogazici University, Istanbul, Turkey

A numerical model for the radio-frequency (RF) ion thruster discharge chamber is developed to simulate the inductively coupled plasma. To simulate real life applications, a transformer model formulation is incorporated with fluid and electromagnetic models. The discharge chamber is assumed to be axially symmetric. Euler equations are modified to include the Lorentz force and forces due to collisions to represent the motion of the plasma. The plasma is treated as a compressible gas. Plasma generation due to ionization is added to the continuity equation. Magnetic vector potential equation is solved for the electromagnetic fields. Spatial distribution of the plasma parameters is obtained. Plasma is assumed as the secondary of an air-core transformer to represent the matching circuit. The resulting model is a tool to be used in thruster performance evaluation.

Nomenclature

$\mathbf{A}$	= magnetic vector potential
$\mathbf{B}$	= magnetic field
$\mathbf{E}$	= electric field
I	= Current
n	= number density
$\mathbf{v}$	= velocity vector field
k	= Boltzmann's constant
l_c	= effective axial length of coil
L_c	= inductance
N	= number of coil windings
$\dot{R}$	= mass generation rate
R	= resistance
P	= Power
PR	= reflected power
$\mathbf{\Gamma}_e$	= electron flux
ν	= collision frequency
Z	= impedance
δ	= skin depth
ω	= plasma frequency
σ	= plasma conductivity

*Graduate Student, Department of Mechanical Engineering, emre.turkoz@boun.edu.tr

†Assistant Professor, Department of Mechanical Engineering, murat.celik@boun.edu.tr

I. Introduction

THE radio-frequency (RF) ion engine, which is also called as RF ion thruster, is an impulse generator. Typical for orbit fine-tuning and station keeping applications in the area of space propulsion, the thrust values generated by these thrusters vary from 10 mN to 200 mN. It is a plasma based device and utilizes the electric field generated by the RF coils to heat up the electrons and the electrostatic force between grids to accelerate the ionized gas out of the plasma chamber. The schematic of a typical RF ion thruster is shown in Fig.1.

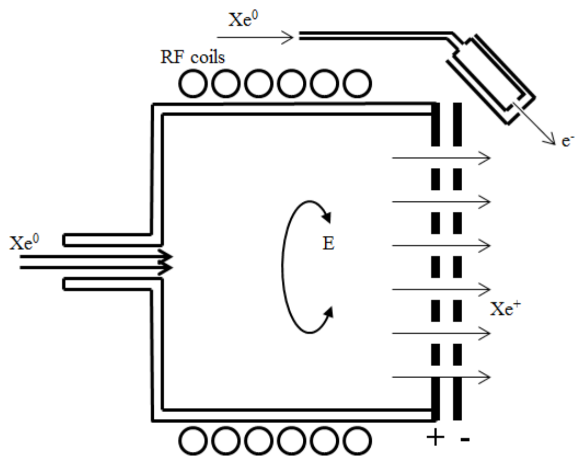

Figure 1. Schematic of an RF ion thruster

The plasma inside the RF ion thruster is inductively coupled plasma (ICP). In order to understand the underlying physics and in the course of digital prototyping through optimization, building a numerical model for the ICP is of great importance. RF ion thruster modeling is an ongoing research area and a complete model is yet to be developed. This work aims to build a numerical model for the ICP in RF ion thruster discharge chamber.

There are recent efforts towards performance evaluation of RF ion thrusters. A leading example[1] is based on evaluating the discharge loss per ion with an analytical model. This 0D model is simple but successful at predicting the performance of ion thrusters in real applications. It lays out the effect of the induced magnetic field due to the RF coils on the ion confinement and therefore decrease in the discharge loss per ion. Another recent 0D model[2] indicates a trade-off between mass utilization efficiency and power transfer efficiency with increasing gas inlet flow rate.

Additional to the analytical models, there are also one or multi-dimensional RF ion thruster discharge chamber modeling studies. A simple transformer model[3] is first laid out for 1D modeling, assuming that the thruster is large enough to assume 1D approach could be valid. Then this model is extended to a 2D model[4] which evaluates the plasma parameters of RF ion thrusters with the help of additional experimental data specific for the thruster to be modeled. In that study, the plasma is treated as a continuum as it is treated in the same way as in this work. There are also studies with the kinetic approach, using a PIC code to solve for the spatial distribution of the plasma parameters. An example model[5] is developed to evaluate the performance of the micro RF ion thrusters.

Using a PIC code is always possible but the usage of the fluid approach decreases the computational cost drastically. Plasma must obey the continuum approach for fluid modeling. The investigation of the question whether the inductively coupled plasma inside the RF ion thruster discharge chamber obeys the continuum approach is already performed[4]. Therefore a fluid model is developed. The model presented in this work consists of three main components: Electromagnetic model, fluid model and the transformer model. The electromagnetic model handles the solution of the Maxwell equations, the fluid model evaluates the motion of the plasma and the transformer model evaluates the matching circuit parameters and most importantly the alternating current magnitude to be supplied to the RF coils. A simpler version of this model is presented previously[6]. That model does not include the treatment of the matching circuit using a transformer model. But the remaining of the model is practically the same.

II. Model

A. Fluid and Electromagnetic Models

The derivation of the equations for the electromagnetic and fluid models is explained elsewhere[6]. Therefore a short summary will be presented here. These electromagnetic and fluid models deal with the parameters that are solved on the 2D axisymmetrical grid. The electromagnetic model consists of the solution of the Maxwell equations using the magnetic vector potential equation:

$$\nabla^2 \mathbf{A} = \mu_0 \sigma \frac{\partial \mathbf{A}}{\partial t} \tag{1}$$

where $\mathbf{A}$ is defined as the magnetic vector potential and formulated as $\nabla \times \mathbf{A} = \mathbf{B}$, and $\nabla \cdot \mathbf{A} = 0$ where $\mathbf{B}$ is the magnetic field. The electric field is also calculated from:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \tag{2}$$

The calculated electric and magnetic fields are incorporated into the fluid equations to calculate the Lorentz force terms. The coupling between the fluid and electromagnetic models is completed with the calculation of the plasma conductivity term to be seen in the magnetic vector potential equation with the electron number density and the temperature values which come from the fluid model.

Fluid equations consist of continuity and momentum equations for the three species: Electrons, ions and neutrals. In addition, electron energy balance equation is solved to calculate the electron temperature distribution. Some assumptions are valid that would help to reduce the complexity of the problem:

- Plasma is quasi-neutral throughout the domain, which means $n_e = n_i$.
- Divergence-free current constraint can be assumed: $\nabla \cdot \mathbf{j} = 0$
- The electron flux can be assumed to obey the drift-diffusion approximation

Using these assumptions, the equations to be solved can be listed as:

Continuity equations:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v_i}) = \dot{R}_i \tag{3}$$

$$\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n \mathbf{v_n}) = -\dot{R}_i \tag{4}$$

Momentum equations:

$$m_i n_i \left(\frac{\partial \mathbf{v_i}}{\partial t} + \mathbf{v_i} \cdot \nabla \mathbf{v_i} \right) + k\nabla(n_i T_i) = e n_i \mathbf{E} + e n_i \mathbf{v_i} \times \mathbf{B} - m_i n_i \nu_{in}(\mathbf{v_i} - \mathbf{v_n}) - m_i n_i \nu_{ei}(\mathbf{v_i} - \mathbf{v_e}) \tag{5}$$

$$m_n n_n \left(\frac{\partial \mathbf{v_n}}{\partial t} + \mathbf{v_n} \cdot \nabla \mathbf{v_n} \right) + k\nabla(n_n T_n) = -m_n n_n \nu_{in}(\mathbf{v_n} - \mathbf{v_i}) - m_n n_n \nu_{en}(\mathbf{v_n} - \mathbf{v_e}) \tag{6}$$

Electron flux:

$$\Gamma_e = n_e \mathbf{v}_e = -\frac{kT_e}{m_e \nu_{eff}} \nabla n_e - \frac{k n_e}{m_e \nu_{eff}} \nabla T_e + \frac{e n_e}{m_e \nu_{eff}} \left(\nabla \phi + \frac{\partial \mathbf{A}}{\partial t} - v_{e,\theta} \times \mathbf{B} \right) \tag{7}$$

Plasma potential:

$$\nabla \cdot (\sigma \nabla \phi) = e \nabla \cdot (n_i \mathbf{v}_i) + \nabla \cdot \left(\frac{ekT_e}{m_e \nu_{eff}} \nabla n_e \right) + \nabla \cdot \left(\frac{ekn_e}{m_e \nu_{eff}} \nabla T_e \right) + \nabla \cdot (\sigma (v_{e,\theta} \times \mathbf{B})) \tag{8}$$

The energy balance for electrons:

$$\frac{3}{2} \frac{\partial}{\partial t} (n_e e T_e) + \nabla \cdot \mathbf{Q_e} = -e \mathbf{E_a} \cdot \mathbf{\Gamma_e} + P_{dep} - P_{coll} \tag{9}$$

So, the fluid model consists of the solutions for the 7 equations given above for the 7 parameters: n_i, n_n, $\mathbf{v_i}$, $\mathbf{\Gamma}_e$, $\mathbf{v_n}$, ϕ and T_e. These parameters represent ion number density, neutral number density, ion velocity, electron flux, neutral velocity, plasma potential and electron temperature, respectively.

The terms $\dot{R}_i$, and $\dot{R}_n$ denote the ion and neutral density generation terms. The terms ν_{in}, ν_{ei} and ν_{en} denote ion-neutral, electron-ion and electron-neutral collision frequencies, respectively. k is Boltzmann's constant, e is elementary electron charge. T_i, T_n, and T_e are ion, neutral and electron temperatures, respectively. P_{dep} denotes the power deposited to the plasma, P_{coll} denotes the power loss due to collisions. $\mathbf{\Gamma_e}$ is the electron flux, and $\mathbf{Q_e}$ is the electron energy flux. $\mathbf{E_a}$ is the ambipolar electric field.

B. Transformer Model

This transformer model is an adaptation of a model presented before[4]. The examples of modeling the inductively coupled discharge as the secondary coil of an air core transformer are to be found also elsewhere[7].

The transformer model is used to update the magnitude of the current supplied to the RF coil. The concept is called the matching network. The coil magnitude is changed to increase the power deposition efficiency. There are models in the literature that evaluates the whole ICP as the secondary oil of an air core transformer and evaluate global plasma parameters. Here in this work, the transformer model is used to evaluate the coil current for maximum power deposition.

In the scope of this model, self-inductance and resistance are attributed to the plasma, which are the properties that normally belong to circuit elements. Then the plasma is handled as depicted in Fig.2. Additional to the plasma properties, the coil properties are also evaluated taking their its geometry into account. The self-inductance of the coil, in [H], which is modeled as a short-solenoid, is expressed as:

$$L_c = 0.002\pi \left(D_w \times 100\right)\left(N^2\right)\left[ln\left(\frac{4D_w}{l_c}\right) - \frac{1}{2}\right] \times 10^{-6} \tag{10}$$

where N is the number of coil windings, l_c is the effective axial length of the coil, D_w is the winding diameter of the coil, which is taken as the outer diameter of the discharge chamber.

The plasma inductance is expressed as:

$$L_p = 0.002\pi(D_p \times 100)\left[ln\left(\frac{4D_p}{L}\right) - \frac{1}{2}\right] \times 10^{-6} + \left(\frac{R_p}{\nu_{en}}\right) \tag{11}$$

where L is the length of the discharge chamber, D_p is the plasma winding diameter in meters, ν_{en} is the electron-neutral collision frequency and R_p is the plasma resistance, which is expressed as:

$$R_p = \frac{2\pi R}{\sigma L \delta} \tag{12}$$

where R is the radius of the discharge chamber, σ is the plasma conductivity and δ is the skin depth. The skin depth is formulated as:

$$\delta = \sqrt{\frac{2}{\omega \mu_0 \sigma}} \tag{13}$$

where μ_0 is the magnetic permeability of free space and ω is the plasma frequency. The mutual inductance is written for a coaxial coil as follows:

$$L_m = 0.0095N\frac{(D_p \times 100)^2}{\sqrt{(D_w \times 100)^2 + (l_c \times 100)^2}} \tag{14}$$

and the transformed plasma inductance and resistance become:

$$L_2 = \frac{-\omega^2 L_m^2 L_p}{R_p^2 + (\omega L_p)^2} \tag{15}$$

$$R_2 = \frac{\omega^2 L_m^2 R_p}{R_p^2 + (\omega L_p)^2} \tag{16}$$

The representation of this model can be seen in Fig.2. The transformed circuit consists of two parallel impedances, called Z_1 and Z_2. The equivalent impedance is called Z.

$$Z_1 = \frac{1}{j\omega C} \tag{17}$$

$$Z_2 = R_c + R_2 + j\omega L_c + j\omega L_2 \tag{18}$$

The total impedance is calculated from the well-known parallel impedances formula

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{\left(\frac{1}{j\omega C}\right)(R_c + R_2 + j\omega L_c + j\omega L_2)}{\left(\frac{1}{j\omega C}\right) + (R_c + R_2 + j\omega L_c + j\omega L_2)} \tag{19}$$

The total impedance is used to characterize the quality of the matching circuit by including this term into the calculation of the power reflection:

$$PR = \left|\frac{Z - Z_0}{Z + Z_0}\right|^2 \times 100\% \tag{20}$$

The percentage of the reflected power is the ultimate parameter needed to evaluate the peak current, which is the magnitude of the AC current passing through the coil. The peak current is calculated as follows:

$$I_{peak} = \frac{\sqrt{2Z_0 P_{forward}}}{Z_0}\left(1 - \frac{Z - Z_0}{Z + Z_0}\right) \tag{21}$$

where $P_{forward}$ is the forward power supplied to the system. The peak current is used to calculate the RMS coil current which will be used to update the current magnitude:

$$I_{coil,RMS} = \left| I_{peak}\left(\frac{Z_1}{Z_1 + Z_2}\right)/\sqrt{2}\right| \tag{22}$$

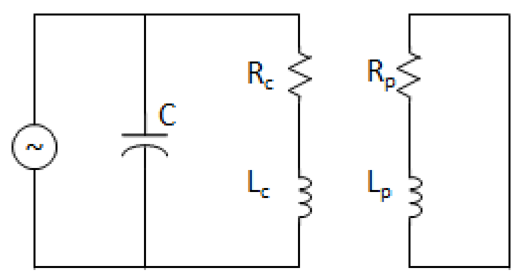

Figure 2. Representation of the transformer model for the ICP.

The coil current calculated by the transformer model enters this formula through the boundary conditions.

III. Solution Technique and Algorithm

Discretization of the momentum equations in axial and radial direction and the continuity equations for the three species are performed using the finite volume method within the SIMPLE algorithm framework[8] for compressible flows[9]. The other partial differential equations are discretized according to the first order central finite differencing scheme. Equations are solved in 2D axisymmetric domain and using cylindrical coordinates. The mesh used in the model is a structured grid with rectangular elements as shown in Fig.3. The time-dependence is handled fully implicitly and the resulting linear systems are solved with ILU-preconditioned GMRES method. The implementation is performed by developing in-house code framework for plasma flow simulations.

The solution starts with an initally assumed uniform plasma conductivity, electron temperature and plasma number density profile. To evaluate the alternating coil current magnitude, the transformer model is initiated. The obtained current is applied as the coil current amplitude to the electromagnetic model to evaluate the EM fields induced inside the discharge chamber. The evaluated fields build the Lorentz force terms in the Euler equations of the fluid model. After the fluid model is solved, the new values for the plasma conductivity is calculated for each node in the mesh. Then the whole process is restarted until the solution converges for each time step.

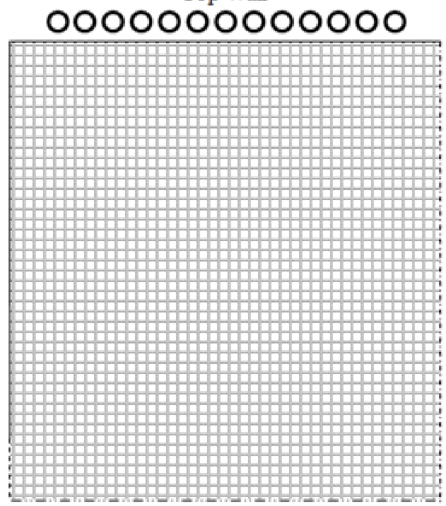

Figure 3. Representation of the grid used in our computations

IV. Results

The thruster configuration tested in the scope of this work is very similar to the RIT-15 ion thruster[10]. The chamber length is 7 cm whereas the diameter is 15 cm. The coil wrapped around the chamber is represented with 13 axisymmetric windings. Inlet neutral flux is assumed to be 2 sccm. The forward power is 400 W. The structured grid is 45x45 whereas the time step is 4.5e-11 seconds. For the results, the term *cycle* is used to denote the time. A cycle denotes the time in seconds that take for an RF cycle to be completed. In this study, the RF frequency is taken as 2 MHz, which corresponds to 5.0e-07 seconds. 50 cycles are completed to ensure the steady state. The code takes 9 hours to complete the 50 cycles on a 3.30 GHz 2-processor Intel Xeon workstation.

The plasma density contour field evaluated is presented in Fig. 4. It is seen that the plasma is confined into the center and diminishes towards the walls. This is due to the fact that ions are lost to the walls. The average plasma density is 3.23e+18 m^{-3}. The same tendency in plasma density distribution is observed also elsewhere[3,5]. Electron temperature is evaluated to be 3.09 eV. This level of temperature indicates that the tail of the Maxwellian causes the ionization in the plasma. We neglect the double ionization in formulating the collision terms in the equations.

The coil current is evaluated using the transformer model. It varies sinusoidally in time and the peak current is calculated at each iteration. The steady state peak current is 3.46 A. Power deposition varies accordingly with the current in time. After steady-state is reached, results indicate that the average power deposition evaluated using the plasma parameters is 403 W, which is very close to the specified 400 W forward power, fluctuating between 107 W and 691 W in one cycle. The power deposition with time for the first 7 cycles is depicted in Fig. 5. The power deposition catches its steady-state pattern after the fourth cycle and continues fluctuating within the limits stated above. The spatial distribution of power deposition into the plasma is presented in Fig. 6. The spatial distribution with cycle ratio indicates that the power deposition penetration is highest when the current is minimum (3/8 and 7/8 cycle ratios), but the maximum amount of power is deposited when the current is also at its peak (2/8 and 6/8 cycle ratios).

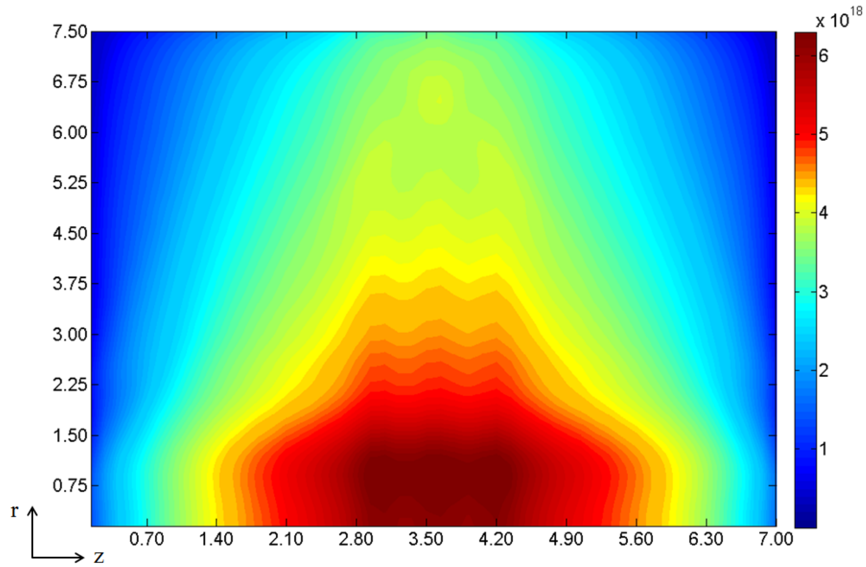

Figure 4. Plasma density contour field for 400 W forward power

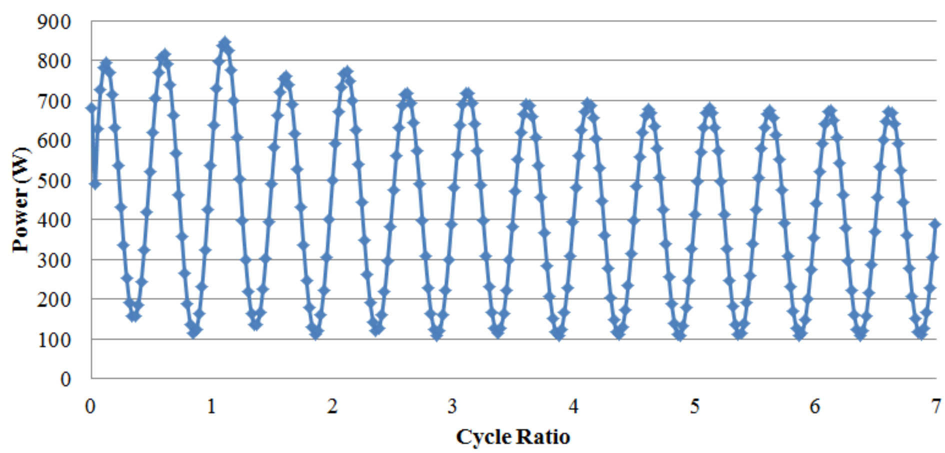

Figure 5. First 7 cycles of power deposition vs. cycle ratio.

V. Conclusion

A numerical model that simulates the ion thruster discharge chamber is developed. The model captures the fluctuations in electromagnetic fields and also represents the matching circuit by simple analytical calculations. Plasma conductivity is calculated using the fluid flow parameters and sent to the electromagnetic model whereas the Lorentz force terms due to the electromagnetic fields affect the flow field to handle the coupling between both models. The transformer model evaluates the required coil current using the plasma parameters averaged over the whole domain. The coil current amplitude is updated at every time step.

It is seen that the forward power we impose to the system is reflected onto the discharge by evaluation of the plasma power through plasma parameters. The fact that the calculated power dissipated, 403 W, is very close to the input power, 400 W, approves that the transformer model is working properly.

The parameter for the ion thruster performance evaluation is the discharge loss per ion, which is the ratio of the total power deposited into the plasma divided by the beam current. In beam current calculations, grid transparency plays a key role. Therefore these parameters should be handled with care. Currently, a constant transparency value is assigned to the grids. The model developed in this study can therefore be further improved by the addition of ion optics physics[11]. This study is aimed to be done as future work.

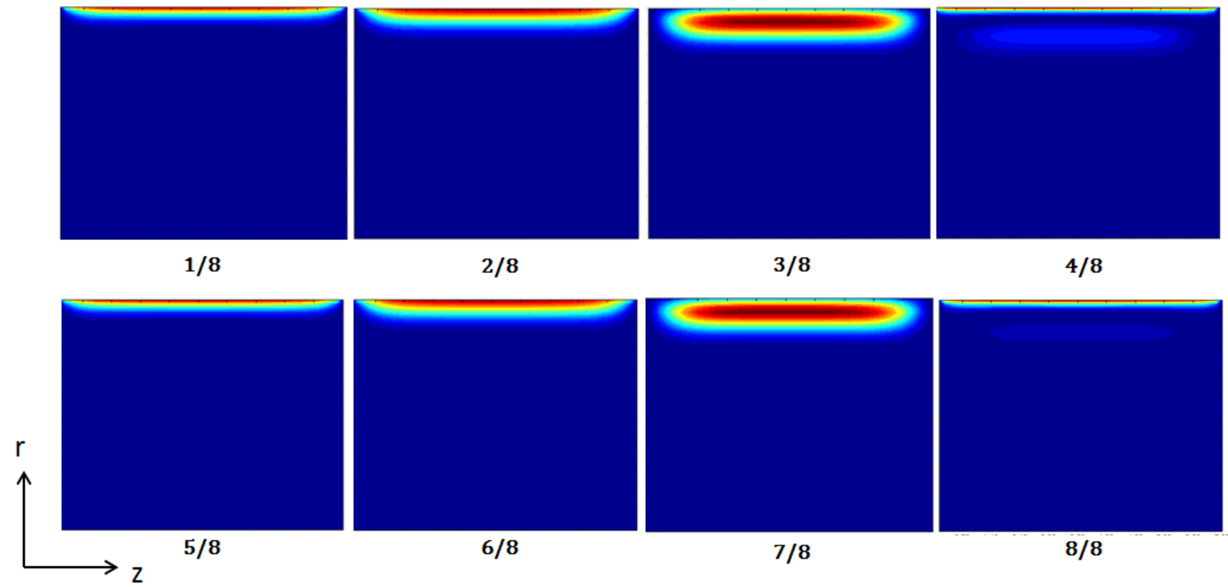

Figure 6. Cycle ratio and power deposition. The fractions denote the cycle ratio within one full RF cycle.

Acknowledgment

This work is partially supported by The Scientific and Technological Research Council of Turkey (TUBITAK) under the projects TUBITAK-112M862 and TUBITAK-113M244. Bogazici University Scientific Research Projects Support Fund (BAP) supports this project under BAP-6184.

References

[1]D. Goebel, *Analytical Discharge Model for RF Ion Thrusters*, IEEE Transactions on Plasma Science, Vol. 36, No. 5, October 2008, pp. 2111-2121.

[2]P. Chabert, J. A. Monreal, J. Bredin, L. Popelier, A. Aanesland, *Global Model of a Gridded-Ion Thruster Powered by a Radiofrequency Inductive Coil*, Phys. Plasmas 19, 073512, 2012.

[3]M. Tsay, M. Martinez-Sanchez, *Simple Performance Modeling of a Radio-Frequency Ion Thruster*, Proceedings to the 30th International Electric Propulsion Conference, Florence, Italy, 2007, IEPC-2007-072.

[4]M. Tsay, *Two-Dimensional Numerical Modeling of Radio-Frequency Ion Engine Discharge*, Ph.D. Dissertation, Aeronautics and Astronautics Dept., MIT., Cambridge, MA, 2010.

[5] Takao, Y., Eriguchi, K., and Ono, K., *Two-Dimensional Particle-in-Cell Simulation of a Micro RF Ion Thruster*, 32nd International Electric Propulsion Conference, Wiesbaden, Germany, September 2011, IEPC-2011-076.

[6]E. Turkoz, M. Celik, *2D Fluid Model for Axisymmetric RF Ion Thruster Cylindrical Discharge Chamber*, 49th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, July 14 - 17, 2013, San Jose, CA, AIAA 2013-4110.

[7]R. B. Piejak, V. A. Godyak, and B. M. Alexandrovich, *A simple analysis of an inductive RF discharge*, Plasma Sources Sci. Technol. 1, 179, 1992.

[8]H. K. Versteeg, W. Malalasekera, *An Introduction to Computational Fluid Dynamics - The Finite Volume Method*, Prentice Hall, 1995

[9]J. H. Ferziger, M. Peric, *Computational Methods for Fluid Dynamics*, 3rd Edition, Springer-Verlag, Berlin, 2002.

[10] H. J. Leiter, H. W. Loeb, and K. H. Schartner. *RIT15S and RIT15LP The Development of High-Performance Mission Optimized Ion Thrusters*, Proceedings of the 35th Joint Propulsion Conference, AIAA-1999-2444. 1999.

[11] J. R. Anderson, I. Katz, D. Goebel. *Numerical simulation of two-grid ion optics using a 3D code*, Jet Propulsion Laboratory, National Aeronautics and Space Administration, Pasadena, CA, 2004.