Numerical Investigation of the Influence of Ambipolar Electric Field on Magnetic Nozzle Detachment

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Abstract: The influence of ambipolar electric field (AEF) on mangetic nozzle detachment is investigated using PIC method. The detachment of electron is highlighted because electrons are easy to be magnetized and difficult to demagnetize and its detachment affects the motion of ions via AEF between them. A hot electron, cold ion and collisionless model is simulated. The model uses the applied magnetic field due to a single loop of current with radius 0.1m centered at the origin. The initial temperature of electron and ion are 40eV and 0, respectively and the strength of the applied magnetic field at the origin is 0.1T. Being different from the numerical simulation using MHD method, a full-PIC method is used. In our simulation, a phenomenon is observed through the distribution of electron number density which we call it "detachment cone" (DC). The following conclusions are drawn. The DC always exists in the magnetic nozzle whether there is AEF or not. But the AEF can influence the shape of the DC and the motion of the electrons in the DC. Through the analysis of demagnetization parameter which is the ratio of the electrons Larmor radius to the scale length of magnetic variation, it can be seen that the AEF can enhance the demagnetization of electron. The AEF begins to influence the detachment of electrons from the position that the non-dimensional axial distance (the ratio of axial distance to the radius of single loop of current) equals 13. It can be explained from the distribution of electric potential well.

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Nomenclature

$\vec{B}$	=	magnetic field
$\vec{E}$	=	electric field
G	=	electron inertia detachment parameter
$\vec{j}$	=	induced current
m	=	particle mass
n	=	number density
r	=	radius
T	=	temperature
u	=	flow velocity
$\vec{V}$	=	particle velocity
Δx	=	grid scale
z	=	z-axis coordinate
β	=	energy ratio of kinetic to applied magnetic field
γ	=	specific heat ratio
ε_0	=	vacuum permittivity
μ_0	=	vacuum permeability
ξ_{demag}	=	demagnetization parameter
ϕ	=	electric potential
Subscript		
e	=	electron
i	=	ion
L	=	Larmor
p	=	plasma
r	=	r-axis
z	=	z-axis
0	=	at the origin

I. Introduction

Electric propulsion is a method that can produce thrust by electrically heating propellant, electrostatically accelerating charged particles, or manipulating the flow of charged particles with electromagnetic fields[1]. Compared with conventional chemical propulsion, one of the main advantages of electric propulsion is its high specific impulse which is suitable for station keeping of telecommunicating satellite in GEO and deep space exploration. According to the mode of energy transforming, electric propulsion can be divided into 3 types: electrothermal propulsion, electrostatic propulsion and electromagnetic propulsion. Recent developments in this discipline have come to incorporate magnetic nozzles for the purpose of plasma flow control in electromagnetic propulsion including Magnetoplasmadynamic thrusters (MPD's), helicon thrusters and Variable Specific Impulse Magnetoplasma Rocket (VASIMR)[1].

Magnetic nozzle is similar to De Laval nozzle. Both of them have a convergent-divergent shape to limit and accelerate the gas or plasma through them. However, the former uses magnetic field generated by solenoid coil whereas the latter uses physical surfaces. Therefore, the physics of magnetic nozzle is complex and is really different from the De Laval nozzle. Recently, plasma detachment mechanisms have become central to magnetic nozzle design to reduce the loss of thrust. In the magnetic nozzle, the plasma needs to detach itself from the imposed magnetic field to form a free plume. Failure to do so would result in a substantial amount of plasma turning around along the magnetic lines and coming back to the spacecraft, ruining efficiency, attacking sensitive surfaces, and polluting the environment of the payload.[2]

In most papers discussing detachment mechanisms, electron detachment study is highlighted. It is clear because electron with relatively small mass is easy to be magnetized but difficult to demagnetize whereas ion with big mass is easy to demagnetize but hard to be magnetized. However, the further purpose to study detachment is as following. In electromagnet propulsion, the thrust mainly depends on the motion of ions with big mass. But ions are limited by electrons with the existence of electric field between them. The detachment of electron is an important factor to the motion of ions, but the mechanisms are complex. Therefore, the research on the detachment of electron is emphasized.

Plasma detachment can be divided into three categories: collisional, collisionless and magnetic reconnection detachment. For collisional detachment, there are two main viewpoints, resistive detachment proposed by Moses et al and recombination detachment proposed by Dimov et al. Resistive detachment is the detachment considering diffusion. The study is driven by the problem that most detachment studies using Magnetohydrodynamics (MHD) method are not ideal because of ignoring diffusion. The conclusion drawn by Moses is that the classical diffusion does no good to detachment, but after the optimization in simulation, it can still promote to the detachment[3]. The recombination detachment mentioned in Dimov's paper focused on the three-particle recombination in magnetic nozzle, not only on the detachment, and an equation of recombination rate is given[4].

Collisionless detachment has been the focus of most research due to the anticipated convergent detachment. The primary means for achieving collisionless detachment are due to loss of adiabaticity, electron inertial effects, and induced magnetic field effects. Among the three means of collisionless detachment, electron inertial effect is the most widely studied. It was first proposed by Hooper who introduced "hybrid" particle which mass, $m_H = \sqrt{m_e M_I}$ to better examine the detachment behavior. It is based on the fact that electron and ion have different inertia (mass) but are connected with each other with the electric field because of quasineutrality[5]. A non-dimensional parameter G, $G \approx \frac{eB_z}{m_e}\frac{eB_z}{M_I}\frac{r_0^2}{u_0^2}$ is introduced to characterize the magnetic inertia. Following his study, Choueiri and Little developed the theory on the momentum transferring[6] and the induced current influence[7]. However, some others criticized Hooper's theory because they pointed out that one of his initial conditions is that the particles are injected without rotational velocity[8]. In this way, the validity of electron inertia effect and the following research is queried. In a word, the detailed mechanisms of detachment are poorly known[9].

It can be seen that the interaction between electrons and ions, i.e., the ambipolar electric field (AEF) makes big differences to the magnetic nozzle detachment, but it cannot be studied directly using MHD. The AEF here means the electric field which results from the different number density distributions of electron and ion caused by their motion differences[10]. Therefore, the way the AEF influences the detachment and the factors will be studied. Furthermore, in our early study, an interesting phenomenon called "detachment cone" in electron number density distribution was observed which will be described in detail in V.B. The relationship among the "detachment cone", the AEF and the detachment will also be studied. It is believed that the study could provide a new point of view to the detachment study.

II. Numerical Simulation Methods

There are several numerical simulation methods for plasma detachment study in magnetic nozzle. One is MHD which is short of Magnetohydrodynamics. The models are usually based on MHD equations for plasma flows. Another is the full PIC which is a conventional methods used to study the physical mechanisms of plasma flows where all particles are treated kinetically. It is a direct simulation technique which models neutrals, ions, and electrons as discrete particles with PIC. Poisson's equation and Maxwell's equations are solved to find the macroscopic electromagnetic field and Newton's second law is solved for individual particle trajectories, as follows[10]:

$$\nabla^2\phi = -\frac{e}{\varepsilon_0}(n_i - n_e) \tag{1}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} \tag{2}$$

$$\nabla \cdot \vec{B} = 0 \tag{3}$$

$$m\frac{d\vec{V}}{dt} = e(\vec{V} \times \vec{B} + \vec{E}), \frac{ds}{dt} = \vec{V} \tag{4}$$

With the PIC method, the electromagnetic field equations are using a finite-difference scheme. A five-point scheme is used for finite differences based on axisymmetric coordinates as

$$\frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{\Delta r^2} + \frac{1}{r}\frac{\phi_{i,j+1} - \phi_{i,j-1}}{2\Delta r} + \frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{\Delta z^2} = -\frac{e}{\varepsilon_0}(n_i - n_e) \tag{5}$$

The value of the two dimensional electric field E_r, E_z on each node is calculated with the relation $E = -\nabla\phi$; a two-point scheme is used to find the gradient as

$$E_z(i,j) = -\frac{\phi_{i+1,j} - \phi_{i-1,j}}{2(z_{i+1} - z_i)} \tag{6}$$

$$E_r(i,j) = -\frac{\phi_{i,j+1} - \phi_{i,j-1}}{2(r_{j+1} - r_j)} \tag{7}$$

The value of applied magnetic field which is static can be calculated by integration whereas the value of self magnetic field can be calculated by iteration as:

$$\vec{B}_s = \nabla \times \vec{A} \tag{8}$$

$$\nabla^2 \vec{A} = -\mu \vec{j} \tag{9}$$

$$B_{s,r} = -\frac{\partial A_\theta}{\partial z}, B_{s,z} = \frac{1}{r}\frac{\partial r A_\theta}{\partial r}, B_{s,\theta} = \frac{1}{r}\frac{\partial A_r}{\partial z} - \frac{1}{r}\frac{\partial A_z}{\partial r} \tag{10}$$

III. Simulation

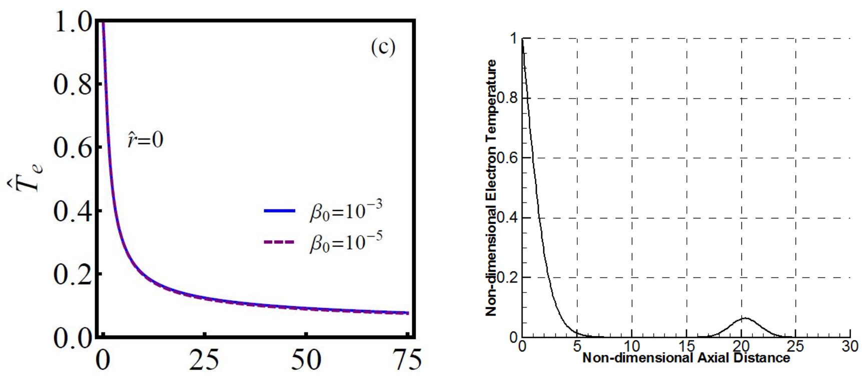
The calculation zone is a 300×200 grids with grid scale Δx=0.01m to simulate the $z \times r$ plate of a cylinder. The models use cylindrical coordinate to calculate. To simplify the problem, the applied magnetic field is produced by a single loop of current with radius a=0.1m centered at the origin and the strength of applied magnetic field at the origin is B_0 =0.1T. There are no electrodes in the zone. The calculation of total magnetic field is composed of calculations of applied magnetic field and self magnetic field. The electric field (AEF) is produced by the diffusion of the electrons and ions. Here the collisions of particles are ignored because it is a far field problem[2]. As the initial conditions of particles mentioned in Ref. 7, the electrons and ions are injected with the initial temperatures of electron, ion and initial plasma radius are $T_{e,0}$=40eV, $T_{i,0}$=0 and $r_{p,0}$=0.018m, respectively.

The simulation can be divided into 2 parts. First of all, a validity demonstration of the numerical simulation program is conducted. Secondly, we study the detailed magnetic nozzle detachment problems.

IV. Results

A. Validity demonstration of the numerical simulation program

This section is a comparison of the variations mentioned in section III.A. "Expansion Properties" in Ref. 7 to verify the validity of the calculation program we used. A comparison of axial variation for electron temperature between Little's work and ours are made for analysis, as shown in Fig. 1. Also, it has to be pointed out that the influence of β_0 is not the key point we study, so we just study the $\beta_0 = 10^{-5}$ case. We get the axial variation data from the two-dimension distribution by using weighted average method, $x_z = \sum_0^r n_{z,r} x_{z,r} / \sum_0^r n_{z,r}$, where x is the variable we study and n is the corresponding particle number density. Similar with Little's work, we use non-dimensional variation which has a "ˆ" on the top of the origin variable, like $\hat{z}$ and $\hat{T}_e$. The non-dimensional definition of axial distance of $\hat{z}$ and electron temperature $\hat{T}_e$ are $\hat{z} = z/a$, and $\hat{T}_e = T_e/T_{e,0}$, respectively.



(a) (b)

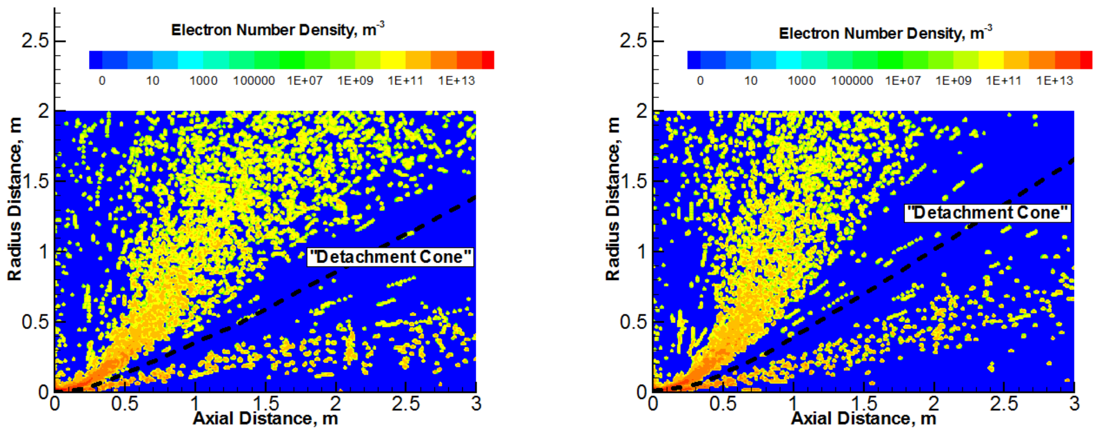
Figure 1. Comparison of axial variation of electron temperature between (a) Little's work and (b) the author's work

In the comparison of electron temperature, the trends of two results are similar with each other. However, in our result, the electron temperature dropped to 0 rapidly whereas in Little's work it is stable around $\hat{T}_e$=0.1. The reason is that the electron number density on z-axis equals zero when $\hat{z} > 7$ in our simulation, as shown in Fig. 1(b). And the flows in MHD method are processed using continuity assumption whereas the particles in PIC method are processed discretely, so the fluctuation at $\hat{z} = 20$ in Fig. 1(b) results from the random distribution of particles in that position, as shown in Fig. 2(a). It can be seen that the maximum of the fluctuation is $\hat{T}_e \approx 0.1$ which is similar with the value in Fig. 1(a) when $\hat{z} = 20$.

B. The influence of AEF to the magnetic nozzle detachment

In our study, an interesting phenomenon is observed, as shown in Fig. 2. We call it "detachment cone" (DC) because in the $z - r$ plate, the curve separates the zone into two parts. The electrons in the part near the axis have a convergent detachment, i.e., they detach closer to the axis. Thus the curve will form a cone in the cylindrical coordinate. Here, we define the convergent detachment zone mentioned above "in" the DC and the other side "out of" the DC. It has to be pointed out that the DC is just a phenomenon observed from the number density distribution figure and there are no strict physical definitions. It proves that the AEF makes no difference to the formation of DC because we simulate the electron motion with and without the existence of ions under the same initial conditions. However, the AEF makes difference to the shape of the DC and motion of electrons in the DC. It can be seen that the electrons in the DC are closer to the z-axis with the AEF than without the AEF. Thus the influence of AEF on the detachment will be discussed in detail later.

In fact, the detachment of electrons is relative to its demagnetization. To the extent to which electrons are demagnetized may be given by the ratio of the electrons Larmor radius to the scale length of magnetic variation. Defining this parameter as ξ_{demag}, we may write $\xi_{demag} = r_{L,e}\frac{|\vec{\nabla} B|}{B}$ where a value of $\xi_{demag} \ll 1$ indicates strongly magnetized electrons, i.e., the ξ_{demag} is larger, the demagnetization is stronger.[7] And



(a) (b)

Figure 2. Electron number density distribution (a) with AEF and (b) without AEF

the condition $\xi_{demag} \ll 1$ is the conservation of adiabatic invariant and the loss of adiabatic will result in the detachment of electrons[1]. Here, the electrons with small mass and small Larmor radius are easily magnetized so they tie to their initial applied magnetic field lines. The ions with big mass and big Larmor radius, however, can wander away from the field line. The AEF between the particles ties the ion to the electron. Thus there is a comparison of axial variation of ξ_{demag} with and without AEF. In Fig. 3, from $\hat{z}$=0∼8, the $\xi_{demag}$with AEF is a little larger than that without AEF when $\hat{z}$ is the same. And from $\hat{z}$=8∼13 the two curves approaches each other. However, when $\hat{z}$ ≥13, with the increasing of $\hat{z}$, ξ_{demag} with AEF continues to increase whereas that without AEF begins to fluctuate and is much smaller than that with AEF. The analysis to this phenomenon is that when $\hat{z}$ ≤13, the effect of applied magnetic field is still strong enough to limit the motion of electrons. Although the AEF potential is increasing and the applied magnetic field is decreasing with the increasing of $\hat{z}$, the effect of AEF is not obvious to the demagnetization. When $\hat{z} \geq 13$, the increasing AEF potential is strong enough to influence the motion of the electron to make it demagnetize. However, the decreasing applied magnetic field is so weak that it makes little difference to the limit of the electron which makes the motion of the electrons unstable.

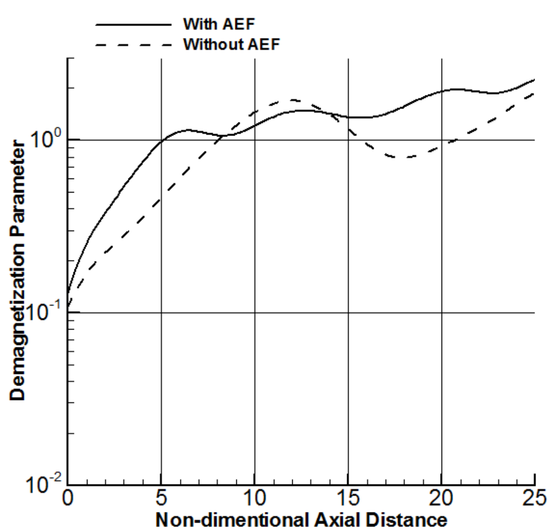


Figure 3. Comparison of axial variation of ξ_{demag} with and without AEF

The question thus arises: how does AEF influence magnetic nozzle detachment? To address this question we get the distribution of electric potential, as shown in Fig. 4. It is clear that the high electric potential near the z-axis results in the electro negativity electrons moving closer to the z-axis. Also, that is the reason how the AEF influences the shape of DC and the motion of electrons in the DC. The distribution of electric potential gives us an intuitive view of the AEF and its influence. From Fig. 4, we also see clearly that the axial range of the region with high electric potential more than 12V is about $10 \leq \hat{z} \leq 25$. And this range is similar with the range $13 \leq \hat{z} \leq 25$ mentioned above in Fig. 3 which explains the influence of AEF well.

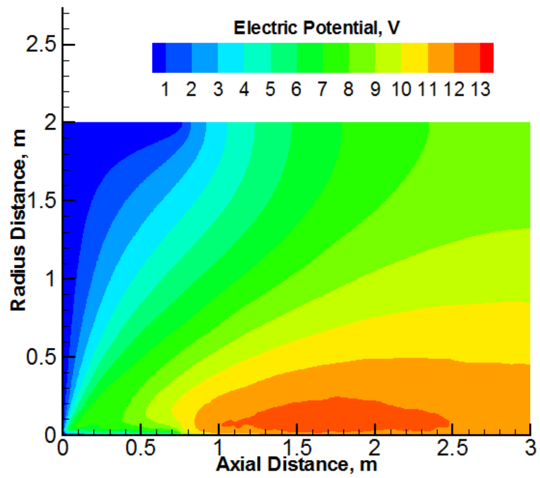


Figure 4. Electric potential distribution

V. Conclusion

A model for the study of detachment of plasma is presented here using the PIC method and the detachment of electron is highlighted because of its strong magnetization and weak demagnetization. What is more, the electrons influences the motion of ions via ambipolar electric field (AEF). The model considers hot electrons and cold ions without collision. Using this model, we examined the influence of AEF on the detachment of electron.

In our study, a phenomenon called "detachment cone" (DC) is observed from the electron number density distribution. The DC always exists in the magnetic nozzle whether there is AEF or not. But the AEF can influence the shape of the DC and the motion of the electrons in the DC. The demagnetization parameter ξ_{demag} is examined to analyze the detachment of the electron. It can be seen that the AEF enhances the demagnetization of electron and begins to influence the detachment of electrons from $\hat{z} = 13$. From the distribution of electric potential, we can see the impact of AEF on the motion of electron more intuitively. Here, the axial range of the region with high electric potential more than 12V is around $10 \leq \hat{z} \leq 25$. And the axial range that AEF works on the detachment of electron through the ξ_{demag} analysis is around $13 \leq \hat{z} \leq 25$. The two ranges are similar.

Acknowledgments

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