Azimuthal oscillations and dynamics of electrons into channel of Stationary Plasma Thruster

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Abstract: In the report represents results of numerical calculation the transport electrons into channel stationary plasma thruster in field azimuthal and longitudinal wave.
For this calculation have been used characteristics wave obtained from experiment.
Results that have been derived by numerical simulation of electron dynamics, analysis Hall parameter, shown, that azimuth wave can increase electron transport velocity along channel thruster.
It is shown, that in elastic collisions in wave field the electrons are displaced along the channel to a higher value, than when driving only in a constant field.
Increasing transport velocity of electrons by interacting them with azimuthal wave can represent through decreasing "scattering time" $\tau_e = \tau_0^2/\tau_i$, where τ_0 – time of scattering electron on neutral atom, τ_i – ionization time.
In taking this in attention Hall parameter have been calculated for two SPD models, operating on Xe and Kr. Comparison this values with experimental values shown, that they enough well coincide between them.

Nomenclature

B	=	radial component of magnetic field
c	=	light velocity
E_w	=	energy of electrons in wave field
e	=	electron charge
f	=	wave frequency
J_i	=	ion current
J_e	=	electron current
k_θ	=	azimuth wave vector
k_l	=	longitudinal wave vector
L	=	wavelength of azimuthally wave
m_e	=	electron mass
n_e	=	plasma density
U	=	potential of electrical field
T_e	=	electron temperature
T_{tr}	=	time of electron transport from exit to anode area in wave field
T_{tr}^0	=	time of electron transport from exit to anode area, waves are absent
V_l	=	phase velocity longitudinal wave
V_d	=	drift electron velocity
V_e	=	electron transport velocity, waves are absent
V_{ew}	=	electron transport velocity in wave field
V_θ	=	phase velocity of azimuth wave
v_e	=	azimuthally electron velocity

v_x = electron velocity along axes x
v_y = electron velocity along axes y
z = axial coordinate direction
β_0 = relative amplitude azimuthally wave
β_l = relative amplitude longitudinal wave
τ_0 = time of scattering electron on neutral atom
τ_i = ionization time
τ_e = effective scattering time
ω_H = electron cyclotron frequency
χ = Hall parameter
σ_w = conduction in azimuth wave
σ_e = conduction no magnetic field

I. Introduction

Currently the plasma stationary thrusters SPT are a leader for performing the correction and the orientation of orbits long-lived satellites. Countries that have space programs, conduct scientific and technical work to improve and expand them a possible applications [1].

There are two main processes that define operation SPT, but not yet fully understood, - is the transport and heating of electrons. They ensure formation of the electric field and the gas ionization in channel thruster.

The classical description of transport (electron scattering on neutral and on the walls of the channel) for the operation of these devices has not found of experimental confirmation [2, 3, 4]. However, studies have shown [3, 5] that in channel SPT and on exit out it there are the significant azimuthal oscillations plasma potential, electron temperature and ion current density, and appear ions, which have the significant azimuthal velocity component. This gave reason to believe [3, 5], that the transfer electrons along channel SPD may be effected by the electron drift in azimuthal electric and radial magnetic fields.

High correlation between oscillation electrical field and plasma density may be defined by ionization processes. Perturbation of velocity electron motion in azimuthal direction (electron drift) can be cause by longitudinal wave of plasma potential. Electron energy which they obtain from wave is accumulated in process elastic collision of electron with neutral atoms. This energy will depend on relation between wavelength of azimuthal wave L and electron collision length. It is clear, this energy will have maximum value if $\tau_0 v_e = L/4$, where v_e is azimuth electron velocity. In order to verify the reliability of the assumptions about the wave heating mechanism and electron transport has been numerically simulated dynamics of electrons in the channel in field azimuthal and longitudinal wave. Since at present there is no clear model of instability, which leads to the appearance of waves, to describe transport and heating electrons, has been used experimentally obtained characteristic of this instability [5].

II. Experimental results

Experimental results have been obtained in process work plasma thruster SPT-70, which are shown on fig.1. These characteristics were obtained when operating on Kr, flow rate was 0.55 cm^3/s, discharge voltage of 200 V, the discharge current of 2.7A.

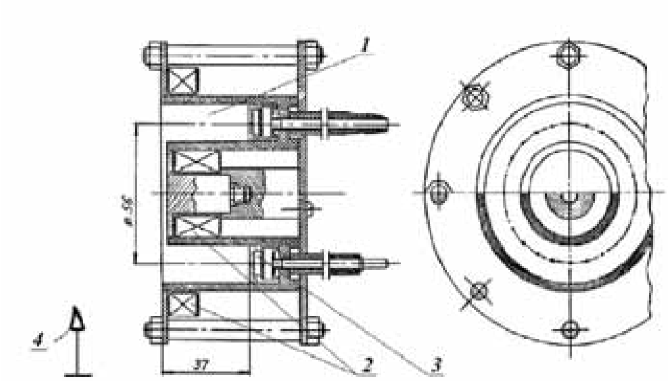


Fig.1. Schematic of stationary plasma thruster SPT 70 (1 - channel acceleration, 2 - magnetic coils, 3 - anode, 4 - electron source)

Investigations of the oscillation in channel have shown that near anode plasma potential oscillations are synchronous. Beginning at z >1.5cm oscillations in range f = (60 – 1000) kHz leads as waves, wave vector which is directed as in azimuth and along channel thruster. Highest relative amplitude is observed near and out exit, where it is reached (30-40)% relatively average value plasma potential. The spectral composition of oscillations, obtained near exit from channel, represented on fig.2. It can seen, that in spectrum is attended several harmonics. The largest amplitudes of the azimuthal waves are observed at frequencies of 60-120 kHz. Similar harmonic oscillations are observed both in the electron temperature and ion current.

Phase velocity of azimuth wave V_θ coincides with direction of electron drift velocity. In band decreasing potential discharge 170-350V V_θ is increased linearly from ~1.3•10^{6}cm/s to ~ 2.4•10^6 cm/s. At the same time is not detected dependency of phase velocity of azimuth wave on magnetic field when his strength is changed in twice. Not detected dependency phase velocity azimuth wave on harmonic frequency in range changing *f* to 1 MHz (not dispersion).

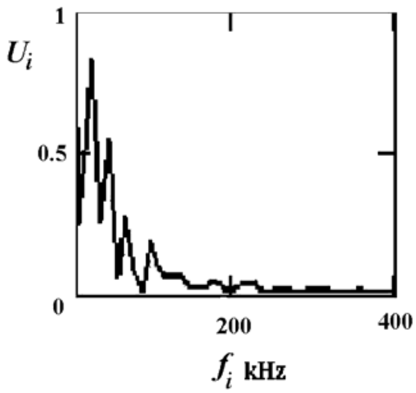


Fig. 2. Spectrum composition of plasma potential oscillation.
U_i - relative amplitude harmonic of oscillation.

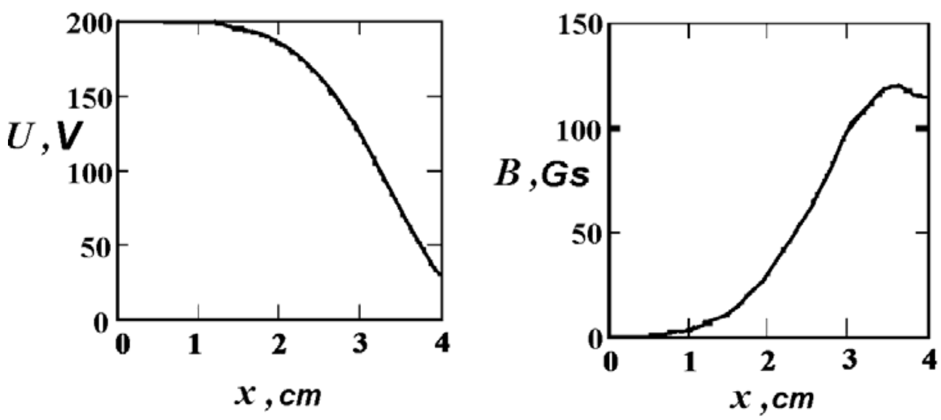
Longitudinal waves have observed in frequency band more 300 kHz. Phase velocity of these waves is not depended of frequency harmonic for f > 400 kHz, but for frequency f <400 kHz significantly is increased with decreasing frequency. Phase velocity longitudinal wave V_l linearly is increased in range (2-5.5)•10^6 cm/s in changing discharge potential from 100V to 350V and is not changed on magnetic field in changing his magnitude more than in twice.

Obviously, the electron temperature oscillations should lead to a change the rate of ionization. Numerical calculations, which are presented in [5], showed that the modulation of ion current in azimuth is the result of oscillations in the electron temperature, the frequency of these oscillations coincides with the frequency of inelastic collisions of electrons with neutrals, leading them to the ionization

A. Numerical simulation of electrons dynamics into thruster channel.

Since at present there is no clear model of instability, which leads to the appearance of waves, to describe transport and heating electrons, has been used experimentally obtained characteristic of this instability [5].

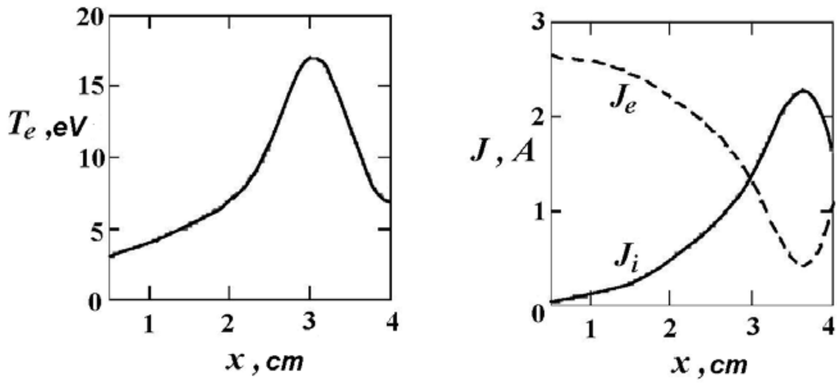
For the numerical calculations we used the following characteristics of the SPT-70 changing along the channel of the electric field $U(x)$ fig.3a, the magnetic field $B(x)$ fig.3b, the ion current $J_i(x)$ fig.3d, the electron current $J_e(x)$ fig.3d and the electron temperature $T_e(x)$ fig.3c. These data were also used to calculate dependence of the collisions time of electrons with neutrals, which lead them to scattering $\tau_0(x)$, and ionization of $\tau_i(x)$. The estimates were made on the assumption that the electron distribution is Maxwellian.



a) electric field

b) magnetic field

Fig.3a, b. Distribution of electrical field *U*, magnetic field *B* in channel thruster
(x=0, anode, x=3.7cm, exit of channel)



c) Electron temperature d) Electron J_e and ion J_i current

Fig. 3c. d. Distribution in channel of electron temperature *Te*, electron J_e and ion J_i current.

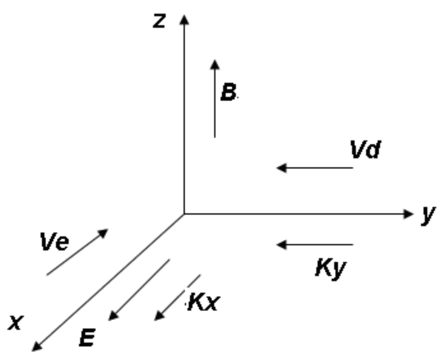


Fig. 4. Scheme fields and electron velocity component,
(*B* - magnetic field, *E* - electrical field, V_d - drift velocity, V_e - electron transport velocity, k_y - azimuth wave vector, k_x - longitudinal wave vector)

System of equations, that describes the electron dynamics with account them inertia in coordinate system Fig. 4 is:

$$\frac{dx}{dt} = v_x;$$

$$m\frac{dv_x}{dt} = -eE(x)(1+\beta_l \sin(\omega_l t + k_l x)) + \frac{e}{c}[v_y B(x)] - \frac{mv_x}{\tau_o};$$

$$\frac{dy}{dt} = v_y;$$

$$m\frac{dv_y}{dt} = -eE(x)\sum_{i=1}^{n}\beta_0 \sin(\omega_0 t + k_\theta y) - \frac{e}{c}[v_x B(x)] - \frac{mv_y}{\tau_o},$$

Calculating was conducted taking into account changes electrical and magnetical fields along channel for value τ_0, k = const.

It is proposed that oscillation amplitude is proportional constant component of electrical field.

Since dispersion is absent $\omega_1 = k_1 v_1$ where k_1, v_1 -wave vector and wave phase velocity in along of channel, $\omega_0 = k_\theta v_\theta$, where k_θ, v_θ -wave vector and wave phase velocity on azimuth in channel.

On fig.5a is represented electron transport from exit to anode, dependency azimuth and longitudinal electron velocity in time when waves is absent and scattering time $\tau_0 = 3 \cdot 10^{-7}$ s.

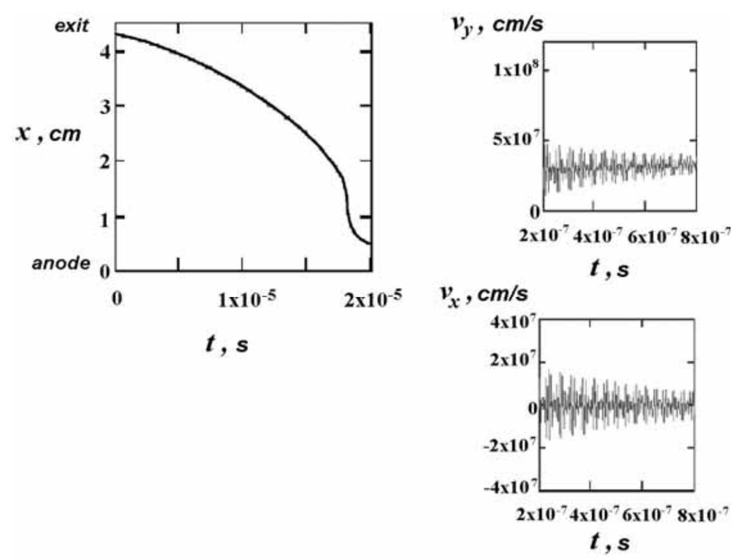


Fig. 5. Transport electron from exit (x=4cm) to anode *x(t)*, dependence azimuthel $v_y(t)$ and longitudinal $v_x(t)$ electron velocities on time for mode, when waves are no.

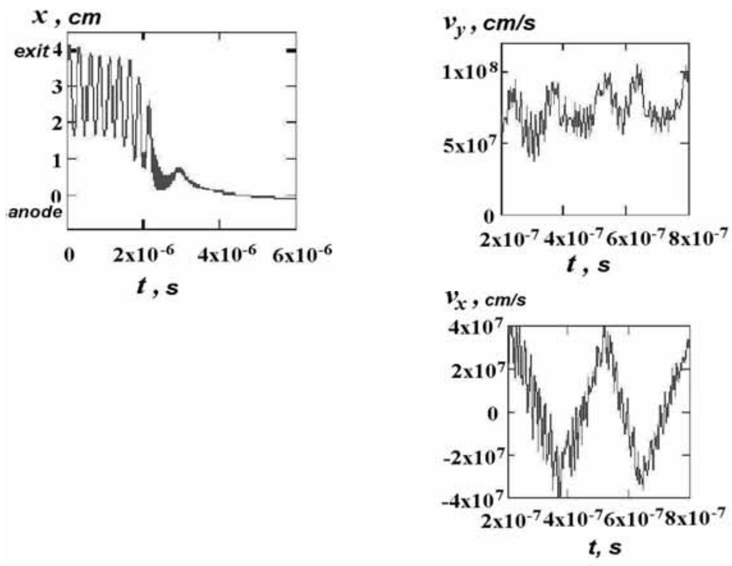


Fig.6. Transport electron from exit (*x*=4cm) to anode *x(t)*, dependence azimuthel $v_y(t)$ and longitudinal $v_x(t)$ electron velocities on time for mode, when waves are in channel.

On fig.5 is represented electron transport from exit to anode, dependency azimuth and longitudinal electron velocity on time when there are waves in channel. For azimuth wave β_0 = 0.35, k_θ =0.3 cm^{-1}, v_θ = 1.5•10^{6}cm/s and longitudinal wave β_l=0.1, k_l =1.4 cm^{-1}, v_l = 3•10^6 cm/s, and scattering time τ_0=3•10^{-7} s.

Such results have been derived in consideration of influence on electron dynamics several oscillation modes specifically k_θ =0.3 cm^{-1}, $k_{\theta 1}$ =0.6 cm^{-1}, $k_{\theta 2}$=1.2 cm^{-1}, β_0=0.12, β_{01}=0.24, β_{02}=0.48.

Numerical calculation shown, if waves there are in channel, electrons transport velocity is increased in several times, arise significant modulation of azimuthal and longitudinal electrons velocity and them of average value are increased fig.5, fig.6.

Estimates show, that period of modulation longitudinal velocity is

$$T_l \approx \frac{2\pi}{(v_d - v_\theta)k_\theta}$$

where v_d velocity electrons drift c•E/H.

Period of azimuthal velocity modulation

$$T_\theta \approx \frac{\pi}{(v_d - v_\theta)k_\theta}$$

In order to identify the functional dependence of the rate of transport of electrons from the wave parameters and frequency of elastic collisions of electrons with neutrals were numerically calculated the time of electrons transfer from the exit $x(4)$ to the anode area $x(1)$ $T_{\rm tr}$. The calculations were performed for a range of changes τ_0, and k_θ, and the amplitude of the oscillations that has been fixed in the channel thruster.

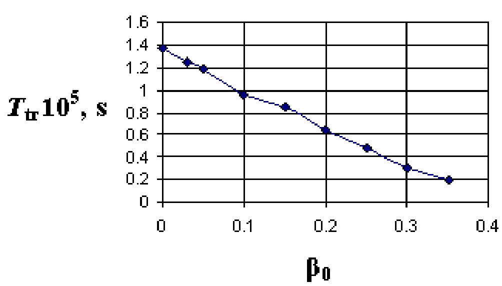


Fig. 7. Changing of transport time of electrons through the channel $T_{\rm tr}(\beta_0)$ on the amplitude of the azimuthal wave β_0 for k_θ=0.35cm^{-1}, v_θ = 1.5•10^6 cm / s, k_l = 1.4cm^{-1}, v_l = 3•10^6 cm/s, β_l = 0.1 and τ_0 = 3•10^{-7}s

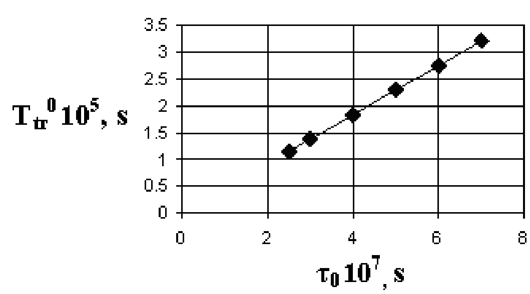


Fig. 8. Changing of transport time of electrons through the channel $T_{tr}^0(\tau_0)$ on the time of electron scattering on neutral τ_0, when the waves are absent

Fig.7 shows the change of transport time of electrons through the channel $T_{\rm tr}(\beta_0)$ on the amplitude of the azimuthal wave β_0 for k_θ =0.35cm^{-1}, v_θ = 1.5•10^6 cm / s, k_l=1.4cm^{-1}, v_l = 3•10^6 cm/s, $\beta_{\rm l}$ = 0.1 and τ_0 = 3•10^{-7}s.

It can be seen that the transport time decreases rapidly with increasing amplitude.

Fig.8 shows the change of transport time of electrons through the channel $T_{tr}^0(\tau_0)$ on the time of electron scattering on neutral τ_0, when the waves are absent.

As can be seen, transport through the channel increases linearly with increasing τ_0, that corresponds to the classical representation of electron transfer in a magnetic field.

Fig.9 shows the change of the time transport of electrons through the channel $T_{tr}(k_\theta)$ on the wave vector k_θ for $v_\theta = 1.5 \cdot 10^6$ cm/s, $\beta_0 = 0.35$, k_l=1.4 cm^{-1}, $v_l = 3 \cdot 10^6$ cm/s, $\beta_i = 0.1$ and $\tau_0 = 3 \cdot 10^{-7}$s.

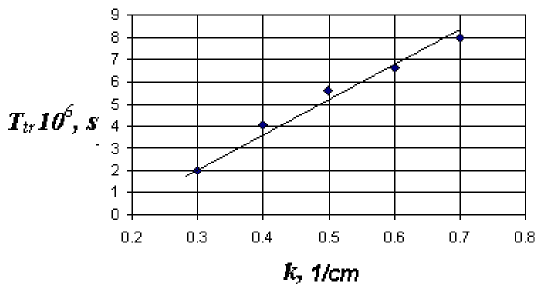


Fig. 9. Changing of transport time of electrons through the channel $T_{tr}(k_\theta)$ on the wave vector of the azimuthal wave k_θ for v_θ = 1.5•10^6 cm/s, β_0=0.35, k_l = 1.4 cm^{-1}, v_l=3•10^6 cm/s, β_i=0.1 and τ_0=3•10^{-7}s.

It can be seen that the time T_{tr} (k_θ) of increases linearly with an increase of the wave vector. As for the considered wave the phase velocity is constant, $\omega_i = v_\theta \cdot k_\theta$ than $T_{tr}(k_\theta) \sim \omega_i / v_\theta$.

Fig. 10 shows the dependence of the ratio of transport time for the classical diffusion and in the azimuthal wave $T_{tr}^0(\tau_0) / T_{tr}(\tau_0)$ on time scattering τ_0 for $k_\theta = 0.25$ cm^{-1}, $v_\theta = 1.5 \cdot 10^6$ cm/s, $\beta_0 = 0.2$, and $k_l = 1.4$ cm^{-1}, v_l=3•10^{6}cm/s, β_l= 0.05.

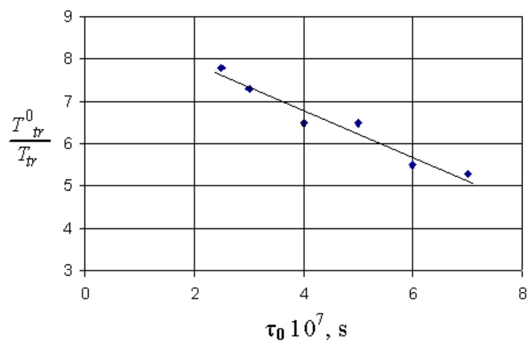


Fig. 10. The dependence of the ratio of transport time for the classical diffusion and in the azimuthal wave T_{tr}^0 (τ_0)/T_{tr} (τ_0) on time scattering τ_0, $k_\theta = 0.25$, $v_\theta = 1.5 \cdot 10^6$ cm/s, $\beta_0 = 0.2$, and $k_l = 1.4$ $v_l = 3 \cdot 10^6$ cm/s, β_l =0.05

As can be seen, this value decreasses linearly with τ_0, $T_{tr}^0(\tau_0)/T_{tr}(\tau_0) \sim 1/\tau_0$, This suggests that the time of transport in the wave field in comparison with classical conductivity increases in proportion to τ_0. Taking this into account, and the fact that $T_{tr}(k_\theta)$ linearly increases with amount the wave vector, dependence of the change time of transport through the channel from the considered parameters can be represented as $T_{tr} \sim \tau_0^2 / \tau_i$, where $\tau_i = 1 / \omega_i$.

That is, the presented results show that the rate of diffusion through the channel depends on the time of electron scattering on neutral and on the parameters of the azimuthal waves.

In taking this into account and the results of numerical calculations of transport time through the channel of τ_0 and k_θ, we can assume, that the rate of electron transfer in the presence of azimuthal wave V_{ew} as compared to the rate of electron transfer V_e for the classical diffusion, is increased in the ratio – $V_{ew}/V_e \sim \tau_i / \tau_0$. As noted, an azimuthal waves, observed in the SPD, has a maximum amplitude at a frequency coinciding with wave frequency, which is determined by ionization processes, then $V_{ew} \sim V_e \tau_i/\tau_0$. Taking this into account, the Hall parameter can be represented as $\chi = \omega_H \tau_0^2/\tau_i$, where $\tau_0^2/\tau_i = \tau_e$ is effective electron scattering time, which decreased in the ionization wave. It should be noted, that longitudinal wave has a much smaller effect on T_{Tr}. It only leads to a modulation of rate v_y.

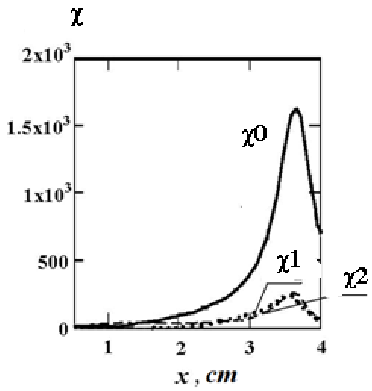


Fig.11. The change the Hall parameter along channel of thruster SPD-70.
χ0 - *classical conductivity,*
χ1 - *experimental*
χ2 - *calculating*

Fig.11 shows the change of the Hall parameter along the channel, where the conductivity is determined only by collisions with neutral **χ0**, the experimental frequency **χ1**, which is obtained from the experiment and in the wave **χ2**=$\omega_H \tau_0^2/\tau_i$. Values τ_0 and τ_i along the channel were obtained by processing the experimental measurements.

As can be seen, the Hall parameter, obtained from the experiment, is in good agreement with the calculated results, which is followed from the analysis of the electron dynamics in the azimuthal wave. This coincidence is appearing for $z > 1.5$ cm, where there is such a wave.

So plasma conduction in azimuth wave could represent, as $\sigma_f = \sigma_e \cdot \frac{1}{1+\omega_H^2 \cdot \tau_f^2}$, where $\sigma_e = e^2 n_e \tau_0 / m$, $\tau_f = \tau_0^2/\tau_i$.

Fig. 12 shows the changes of the Hall parameter along the length of the channel for SPT-130 models, working gas Xe, flow rate 6.3 mg/s, the discharge voltage 200B, discharge current of 6.7A. Method of calculation time of electron collisions with Xe τ_0, τ_i, and ω_H along the channel was used the same as for the SPT-70. As seen, the Hall parameter obtained by processing the experimental results χ1(*x*), agrees quite well calculation with results χ2(*x*), which was obtained when considering transport in field azimuthal wave for this plasma thruster.

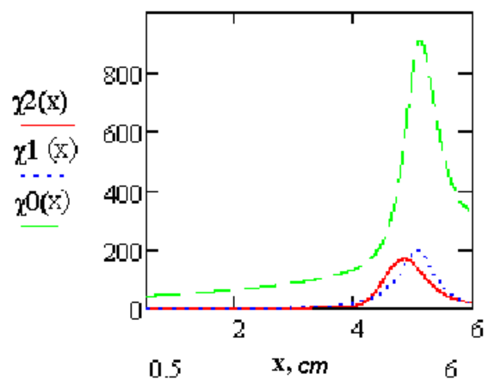


Fig. 12. ***The change the Hall parameter along channel of thruster*** **SPT- 130**
χ0(x) - for *classical conductivity*,
χ1(x) - *experimental results,*
χ2(x) - *calculating*

Discussion of the results

Thus results, obtained in numerical simulation dynamics electrons in wave field, analysis of Hall parameter, gives possibility to believe that azimuth wave ensures transport electrons in channel SPT. Estimations have showed that effective time of electrons scattering by interacting them with azimuthal wave τ_e can represent as $\tau_e = \tau_0^2/\tau_i$. Longitudinal wave has a much smaller effect on T_{Tr}. It only leads to a modulation of rate v_y.

Qualitatively, this can be represented as follows. In the case where there is the azimuthal wave electron velocity increases as it follows from the numerical calculations. As far as drift velocity is much greater than the phase velocity, the movement of electrons in the wave field can be considered in a quasi-stationary approximation. Consequently, their energy E_w should increase to maximum value if the mean free path between collisions

$\tau_0 \bullet v_y = L \bullet n/4$, where n = 1, 3, 5 and decrease if $\tau_0 \bullet v_y > L \bullet n/4$, where n = 1, 3, 5. If $\tau_0 \bullet v_y = n / 2 \bullet L$, where n = 2, 4, E_w=0. Obviously the electron velocity will depend on the expansion of the wave field, if $\tau_0 \bullet v_y < L \bullet n/4$, then $v_{ef} \sim eE\tau_0/m_e$.

Thus, if the energy of the electrons will increases, then the collisions their with neutrals should also increase their average displacement along channel [6]. The growth of energy will depend on the relationship between the frequency of collisions electrons and the frequency of the azimuthal wave. Obviously, the energy of the electrons in the wave field will gain greater with increasing wavelength.

However, it should be noted, that absence of a model of instability inducing to oscillations of this type, does not allow to fully carry out a detailed analysis of electron diffusion, based only on experimental results, since it is not clear what determines the amplitude and the spectral composition of oscillations, while from them depend the amount of diffusion and heating of electrons.

In calculating is proposed that electron function distribution is Maxwellian, however need point out, that really this function may be different from it.

III. Conclusion

To test idea about wave mechanism transport electrons in thruster channel has been numerically simulated electron dynamics in field azimuth and longitudinal wave. Since at present there is no clear model of instability, which leads to the appearance of waves, to describe transport and heating electrons, has been used experimentally obtained characteristic of this instability.

These calculating showed, that azimuth waves leads to increasing electrons transport velocity, as comparison to classical diffusion, which is defined by scattering electros on neutral atom. This is due to the fact that in wave field the azimuth velocity of electrons would increase significantly compared with the rate of drift velocity in a constant field. Then in elastic collisions in wave field the electrons are displaced along the channel to a higher value, than when driving in a constant field. Electron energy which they obtain from wave is accumulated in process collision of electron with neutral atoms, their energy E_w should increase to maximum value if the mean free path between collisions $\tau_0 \bullet v_y = L \bullet n / 4$, where n = 1, 3.

Analysis of frequency characteristic this wave, modulation ion current density and electron temperature give them possibility to propose that this wave is effected by ionization instability.

It was showed that time of electron scattering by interaction them with azimuth wave is decreased and it is described well by relation $\tau_e = \tau_0^2/\tau_i$.

Plasma conduction in field azimuthally wave may be represented, as $\sigma_f = \sigma_e \cdot \frac{1}{1+\omega_H^2 \cdot \tau_f^2}$, where $\sigma_e = e^2 n_e \tau_0 / m$, $\tau_f = \tau_0^2 / \tau_i$.

Comparison Hall parameter that have been produced from this representation for the two models SPT operating on different gases Xe and Kr are in good agreement with the experimental results.

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