Generation of Microwave Range Electromagnetic Radiation in the Hall Thruster Plasma

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Sergey A. Oghienko[1]
National Aerospace University "Kharkiv Aviation Institute", Kharkiv, 61070, Ukraine

Abstract: A ways of generation of microwave range electromagnetic radiation (MWR) in Hall thruster (HT) plasma, which is interference to a communication radio signal of a space vehicle, are investigated. Results of experiments on MWR power spectral density determining of researchers from the USA (Beiting E.J. and others) and Russia (Kirdyashev K.P. and others) are used for the analyzing in this paper. Basing on these data it was estimated possible range of MWR power in characteristic frequency range f=1.5…10 GHz in dipole approximation and it was analyzed next probable ways of MWR generation. 1) Braking and 2) Cyclotron radiation. 3) Radiation of electrons accelerated by electric field in radial direction in plasma boundary layer potential drop. 4) Radiation of electrons accelerated in an azimuthal direction (by local electric field generated because of azimuthal heterogeneity of charge) in volume of a plasma stream. 5) Transformation of mainly longitudinal (plasma) waves, generated in plasma because of Vavilov-Cherenkov effect (VCE) into MWR of power up to 10^{-3} W and frequency $f \leq 5$ GHz in a layer of smoothly heterogeneous plasma concentration out of HT discharge chamber. Last of them - 4) and 5) - are most probable reasons of MWR generation in HT plasma.

Nomenclature

B, B_r = magnetic field induction radial component
E = electric field intensity
E_{loc} = local electric field intensity
f = wave frequency
m = mass of electron
$n_{e,i}$ = concentration of electron, ion
R_{gyro} = electron gyro-radius
S = square of surface
T_e = electron temperature
$V_{.tr}$ = electron cross-field transportation velocity
W = radiated microwave power
w = spectral density of an energy flow
ε = permeability of plasma
φ = electric potential
ω = circular frequency

I. Introduction

THE base design of the HT is used in spacecraft propulsion units already tens years. For further development of HT technologies it is necessary to solve problems, which limits use of HT advantages. One of such problems is

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[1] PhD, Senior Researcher, Department of thrusters and power plants of space vehicle, oghienko@yahoo.com

a radio noise (the microwave radiation from HT plasma) for operation of spacecraft send-receive radio devices. To find the technical decision of this problem it is necessary to understand a role of the basic processes in plasma, which determine power and frequency of MWR, generated in HT plasma. This research was carried out to reveal the most significant processes of charge movement, which can cause generation of MWR in HT plasma.

II. Problem analysis

It is known that a frequency range of MWR, generated in HT plasma, is crossed with a range of a spacecraft radio signal $f_{sign}>2\ldots10$ GHz, used for communication through an Earth ionosphere. Power of MWR from HT plasma many times over surpasses power of a communication radio signal that makes impossible the uninterrupted information transfer. Power of MWR from SPT M-100 type is estimated basing on known experimental data about MWR spectral density as follows.

Results of experiments in Russia - Kirdyashev K.P. and others[1, 2, 3, 4] on MWR from HT plasma in a range of frequencies $f\approx1.5\ldots10$ GHz at peak spectral density of an energy flow $w_{exp}\approx10^{-8\ldots-7}$ W/(m^2·MHz) in a band of $f\approx1.5\ldots2.5$ GHz (width $\Delta f\approx10^3$ MHz) through a hemisphere area S_R are known. Basing on these data, I estimate power W_{exp} of MWR from HT plasma as $W_{exp}=w_{exp}\cdot\Delta f\cdot S_{Rw}\cdot1/2$, where area of receiving radiation S_{Rw} was estimated in the next way. Unfortunately, in experimental researches mention above, it was only specified an arrangement of the receiving antenna - at a wall of the vacuum chamber. Basing on this, I have estimated area of receiving radiation as $S_{Rw}\approx4\cdot\pi\cdot1^2/2\approx6$ m^2, assuming radius of the vacuum chamber – 1≈1 m. So full power was estimated as $W_{exp}=w_{exp}\cdot\Delta f\cdot S_{Rw}\cdot1/2\approx10^{-4\ldots-3}$ W. In other research, as it have been shown by authors in scheme of experiment, the receiving antenna was installed close to the thruster and, so, receiving radiation area is $S_{Rn}\approx50\cdot10^{-4}$ m^2, and MWR power is estimated as $W_{exp}=w_{exp}\cdot\Delta f\cdot S_{Rn}\cdot1/2\approx10^{-8\ldots-7}$ W. Thus, basing on experimental data of researchers from Russia it is estimated MWR power possible range as $W_{exp}=10^{-8\ldots-3}$ W.

Experimenters from the USA - Beiting E.J. and others (Ref. 5) - registered peak spectral density of an energy flow $w_{exp}\approx10^{-4\ldots-3}$ W/(m^2·MHz) in a band of 3…5 GHz (width $\Delta f\approx(5\ldots3)\cdot10^3$ MHz) on distance of 1 m from the HT in a forward hemisphere of area $S_P\approx4\cdot\pi\cdot1^2/2\approx6$ m^2. Basing on this results, I have estimated power W_{exp} of MWR, living HT plasma in a band $f\approx1.5\ldots10$ GHz as $W_{exp}=w_{exp}\cdot\Delta f\cdot S_P\cdot1/2\approx1\ldots0.1$ W.

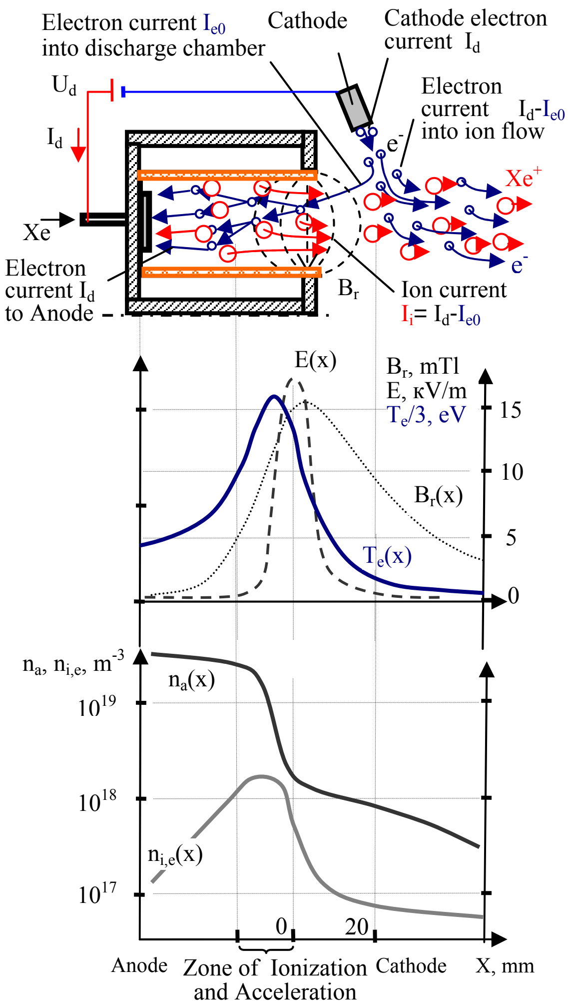


Figure 1. Distribution of plasma flow parameters, fields in HT discharge interval. $n_{i,e}$(x) and T_e(x) were taken close to the plasma flow boundary.

Now results of researches executed in Russia, in which two ways of MWR generation were analyzed, are known - for example[2, 3, 4]. These ways are: 1) transformation of plasma waves into electromagnetic waves; 2) MWR generation by plasma noise fluctuations strengthened by plasma waves. This result was confirmed by author's calculation at spectral density of an energy flow $w_{exp}\approx10^{-8\ldots-7}$ W/(m^2·MHz) (in a range of MWR power $W_{exp}\approx10^{-8\ldots-3}$ W) and frequency $\omega_{exp}\approx12$ GHz, whereas MWR power range experimentally determined in USA is even up to $W_{exp}\approx1\ldots0.1$ W in frequency range $\omega_{exp}\approx9\ldots60$ GHz and more.

Basing on the analysis of the data mentioned above in view of HT plasma characteristic parameters (look fig. 1) shown in Ref. 6, it was curried out analyze of the reasons of MWR power generation in band 0.1…1 W, $10^{-8\ldots-3}$ W and in range of frequencies $\omega_{exp}\approx9\ldots60$ GHz as: 1) direct result of acceleration of charges in plasma; 2) as result of plasma waves transformation into electromagnetic waves or plasma noise fluctuation strengthening. In this paper power and frequency of MWR, generated in HT plasma owing to various reasons, are estimated as follow.

III. Braking radiation

Estimation of electron braking radiation power is curried out in dipole approach, following methodic Ref. 7. For the simplification it was supposed the next. Distribution of electrons in space of velocities is δ-function and velocity of all electrons meet most probable velocity of Maxwell distribution $V=V_{prob}=(T_e \cdot e \cdot 2/m)^{1/2}$ at temperature - T_e (eV unit), electron mass $m \approx 9 \cdot 10^{-31}$ Kg and value of an elementary charge $e \approx 1.6 \cdot 10^{-19}$. Designations and characteristic values are entered: a range of MWR frequencies is $\Delta\omega \approx 2 \cdot 10^{9}$ Hz; plasma characteristic concentration is $n_i \approx n_e \approx 5 \cdot 10^{17}$ m^{-3}; electron temperature $T_e \approx 30$ eV and $V(T_e \approx 30) \approx 3 \cdot 10^{6}$ m/s; V_{pl} - volume of plasma, in which MWR is generated, is a site of a plasma stream (in a discharge interval where the magnetic induction and an electric field are significant) with the area of cross-section $S \approx 50 \cdot 10^{-4}$ m^2 and extent $L \approx 4 \cdot 10^{-2}$ m so, that value $V_{pl} \approx S \cdot L \approx 2 \cdot 10^{-4}$ m^3; dielectric permeability of plasma $\varepsilon \approx 1-\omega_L^2/\omega^2 \approx 0.5$, ω_L – Langmuir (plasma) frequency; velocity of light $c=3 \cdot 10^{8}$ m/s, Euler's constant $\gamma=1.78$. Then power W_T of electron braking radiation is calculated, following formulas Ref. 7 as

$$W_B = \sqrt{\varepsilon} \cdot \frac{16 \cdot e^6 \cdot n_i}{3 \cdot V \cdot c^3 \cdot m^2} \cdot \ln\left(\frac{2 \cdot m \cdot V^3}{\gamma \cdot \omega \cdot 2 \cdot e^2}\right) \cdot \Delta\omega \cdot n_e \cdot V_{pl} \approx 10^{-21}\ \mathrm{W},$$

where: electron absolute velocity $V(T_e \approx 30) \approx 3 \cdot 10^{6}$ m/s, V_{pl} - volume of plasma space, a range of MWR frequencies $\Delta\omega \approx 2 \cdot 10^{9}$ Hz.

Corresponding circular frequency of radiation is estimated as

$$\omega \leq (m \cdot V^3 \cdot 4 \cdot \pi \cdot \varepsilon_0)/e^2 \approx 7.5 \cdot 10^{16}\ \mathrm{Hz},$$

that is essentially more than experimentally registered values. To calculate power W_B of radiation close to frequency $\omega_{exp} \approx 15$ GHz (it is registered in experiments) it is necessary to consider the contribution of electrons with velocities $V \leq 1.8 \cdot 10^{4}$ m/s $<< V(T_e \approx 30) \approx 3 \cdot 10^{6}$ m/s. In HT plasma the share of such electrons is small - within several percent and, hence, W_B ($V \leq 1.8 \cdot 10^{4}$ m/s) $<< W_{exp}$.

Thus, electron braking radiation cannot cause MWR from HT plasma even $W_{exp} \approx 10^{-8 \ldots -3}$ W in circular frequencies $\omega_{exp} \approx 9 \ldots 60$ GHz, which are observed in experiment.

IV. Cyclotron radiation of electrons

Harmonics of cyclotron radiation frequency, which correspond to known experimental results ($\omega_{exp} \approx 9 \ldots 60$ GHz), and EMR power from HT plasma radiated in these harmonics, are determined. Characteristic parameters of HT plasma are next: electron temperature $T_e \approx 30$ eV, charge concentration $n_e \approx 5 \cdot 10^{17}$ m^{-3}, intensity of electric field $E \approx 10^{4}$ V/m, magnetic field induction $B \approx 15$ mTl in the area of radiation - in volume $V_{pl} \approx 2 \cdot 10^{-4}$ m^3 of plasma flow.

Power of cyclotron radiation in m^{-th} harmonic from volume of plasma V_{pl} with concentration n_e was calculated following methodic Ref. 7, 8 as

$$W_{Cm} = \frac{e^2 \cdot \omega_{Cl}^{\ 2}}{2 \cdot \pi \cdot \varepsilon_0 \cdot c} \cdot \frac{(m+1) \cdot m^{2 \cdot m+1}}{(2 \cdot m+1)!} \cdot \beta_{\perp}^{\ 2 \cdot m} \cdot V_{pl} \cdot n_e .$$

As frequency of the 1^{-st} (m=1) harmonic is $\omega_{Cl}=e \cdot B/m \approx 2.7$ GHz, the only 4^{-th} harmonic (m=4) of frequency of cyclotron radiation in a magnetic field meets to experimental results ($\omega_{exp}/\omega_{Cl} \geq 4$) so that $4 \cdot \omega_{Cl} \geq \omega_{exp \cdot min}=9$ GHz ($f_{exp \cdot min} \approx \omega_{exp \cdot min}/2/\pi=1.5$ GHz). In 4^{-th} harmonic (m=4) at $\beta_{\perp}=V_{\perp}/c \approx 6.3 \cdot 10^{-3}$ (electron velocity component $V_{\perp}$, which is perpendicular to a magnetic field, was estimated as $V_{\perp} \approx (T_e \cdot e \cdot 2/3 \cdot m)^{1/2}$) the value of cyclotron radiation power $W_{Cm=4} \approx 4 \cdot 10^{-15}$ W $<< W_{exp} \approx 10^{-8 \ldots -3}$ W. In harmonics of more than m=4 the power of radiation decreases as $\sim \beta_{\perp}^{2 \cdot m}$ where $\beta_{\perp} << 1$.

V. Microwave radiation of electrons accelerated in a plasma boundary layer

It was calculated MWR frequency ω_{BL} and power $\mathbf{W_{BL}}$, radiated by separate electron accelerated periodically in a plasma boundary layer, in dipole approach following methodic Ref. 8. And then it was calculated full power of

MWR from area of a boundary plasma layer, accounted value of electron flow from internal plasma volume to this layer.

To estimate full power of MWR, next plasma parameters (on border with a dielectric chamber wall) that are characteristic for plasma of HT discharge interval were chosen basing on results of experimental researches shown in Ref. 6. Thickness of the layer that lock electrons in plasma is estimated as $\delta \approx 4 \cdot 10^{-4}$ m (look fig. 2) and locking potential drop is estimated as $\Delta\varphi_{BL} \approx 2.5 \cdot T_e$ (in view of secondary electron emission Ref. 6). Then the time period τ_{BL} of electron periodic movement through a layer of thickness δ with electron velocity before plasma-layer border $V_{eT} \approx (T_e \cdot e \cdot 2/m)^{1/2} \approx 2 \cdot 10^6$ m/s (at temperature $T_e \approx 30$ eV) is estimated as $\tau_{BL} \approx 2 \cdot \delta / V_{eT}$. The circular frequency ω_{BL} of MWR, which correspond to this periodic electron movement with acceleration and deceleration, is estimated as $\omega_{BL} \approx 2 \cdot \pi / \tau_{BL} = \pi \cdot V_{eT} / \delta \approx 15 \cdot 10^9$ Hz.

MWR power of single electron, accelerated up to $a \approx \Delta\varphi_{BL} / \delta \cdot e/m$ in a layer of thickness δ with potential drop $\Delta\varphi_{BL}$, is calculated as $w_e = (a \cdot e)^2 / (6 \cdot \pi \cdot \varepsilon_0 \cdot c^3)$ in dipole approach, following methodic Ref. 8. Then energy E_τ, radiated for the period τ_{BL} of electron movement through a boundary plasma layer, is calculated as $E_\tau = w_e \cdot \tau_{BL}$.

MWR power from a boundary plasma layer in the discharge chamber was calculated as $W_{BL} = E_\tau \cdot S \cdot n_e \cdot V_{eT}$, where: the area of a boundary plasma layer was calculated as $S \approx \pi \cdot (D_{ext} + D_{in}) \cdot L_{ZIA}$, where diameter of external wall is $D_{ext} = 100$ mm and diameter of internal wall - $D_{in} = 70$ mm, axial extent – $L_{ZIA} = 10$ mm, concentration of electrons – $n_e = 5 \cdot 10^{17}$ m^{-3} the density of an electron flow to a layer surface is of the order $n_e \cdot V_{eT} \approx 5 \cdot 10^{17} \cdot 2 \cdot 10^6 \approx 10^{24}$ $1/(s \cdot m^2)$. Than MWR power is $W_{BL} = E_\tau \cdot S \cdot n_e \cdot V_{eT} \approx 10^{-8}$ W.

Analyzing of characteristic plasma parameters distribution plotted in fig. 1 and 2, it was resulted that: 1) thickness of a boundary layer plasma-vacuum more than thickness of a layer plasma-dielectric (look fig. 2) as concentration of charges in a plasma flow outside discharge chamber less than concentration in the discharge chamber because of plasma flow expansibility; 2) potential drop in a boundary layer plasma-vacuum essentially less than in a boundary layer plasma-dielectric as temperature of plasma in a flow outside discharge chamber essentially less than in the discharge chamber. As a result of these conditions the MWR power from a plasma-dielectric layer is greater than radiation from a plasma-vacuum layer.

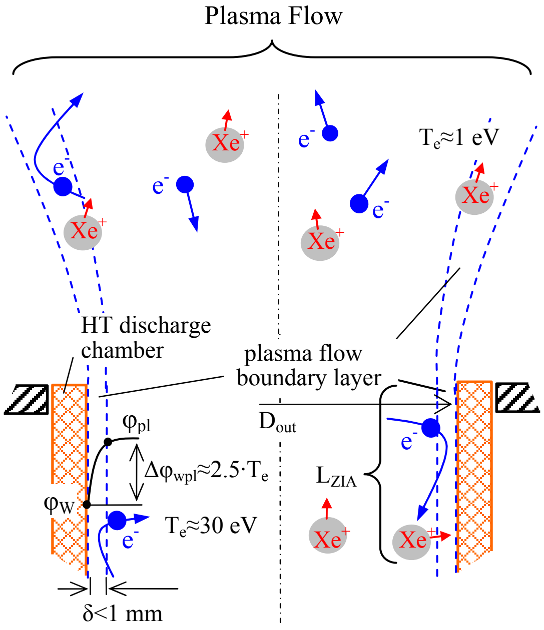


Figure 2. Periodical movement of electrons through plasma flow boundary layer.

Thus, MWR power $W_{BL} \approx 10^{-8}$ W in frequency $\omega_{BL} \approx 15 \cdot 10^9$ Hz, generated owing to periodic acceleration of electrons in a plasma boundary layer, is close to the bottom MWR power threshold determined experimentally (on data from Russia $10^{-8 \ldots -3}$ W). Radiation in frequencies greater than $\omega_{BL} \approx 15 \cdot 10^9$ Hz can caused by electrons, which penetrate into plasma boundary layer less than depth δ.

VI. Macrowave radiation of electrons accelerated in an azimuthal potential heterogeneity

In HT discharge chamber it is existed azimuthal heterogeneity of gas concentration. As consequence of this, because of gas ionization, azimuthal heterogeneity of ion density is generated. Because of azimuthal heterogeneity of ion density and azimuthal homogeneity of electrons, drifted in crossed electric and magnetic fields, it is existed uncompensated ion positive charge. As consequence of uncompensated charge, it is generated azimuthal electric field. This uncompensated charge of ions is compensated in the next way. Local areas of extent $l_h \sim (0.1 \ldots 1) \cdot R_{le} \approx R_{le}/2$ with potential drop $\Delta\varphi_h \leq 1$ V in an azimuthal direction (look fig. 3) are generated in plasma. Because of dispersion of drifting electrons in these local areas by electric field $\approx \Delta\varphi_h / l_h$ the average velocity of electron azimuthal drift in these areas is decreases. As consequence of this, it is occurred compensation of ion charge azimuthal heterogeneity by electrons. Electrons, drifting with constant velocity $V_{edr} = E/B$ in crossed electric E and magnetic B fields in an azimuthal direction and passing through potential drop $\Delta\varphi_h$, are accelerated/decelerated periodically. As a result, electromagnetic radiation is generated and leave plasma without essential losses. Frequency of such radiation is determined by a ratio $\omega_h \approx 2 \cdot \pi \cdot V_{edr} / l_h \approx 10^{10 \ldots 11}$ Hz, where characteristic electron drift velocity is $V_{edr} \approx 2 \cdot 10^6$ m/s.

MWR power of electrons, moving with acceleration $a \approx \Delta\varphi_h / l_h \cdot e/m \approx 1.6 \cdot 10^{14}$ (m/s^2) through potential drop $\Delta\varphi_h$ in local area, is determined as $w_h = (a \cdot e)^2 / (6 \cdot \pi \cdot \varepsilon_0 \cdot c^3) \approx 2 \cdot 10^{-25}$ W in dipole following methodic Ref. 8. Space, in which MWR radiation is mainly generated, is a reference volume of a plasma flow in discharge interval (where magnetic induction and electric field are significant) with the area of cross-section $S \approx b_k \cdot D_{av} \cdot \pi \approx 50 \cdot 10^{-4}$ m^2 (look fig. 3), extent $L \approx 4 \cdot 10^{-2}$ m and volume of the order $V_{pl} \approx S \cdot L \approx 2 \cdot 10^{-4}$ m^3.

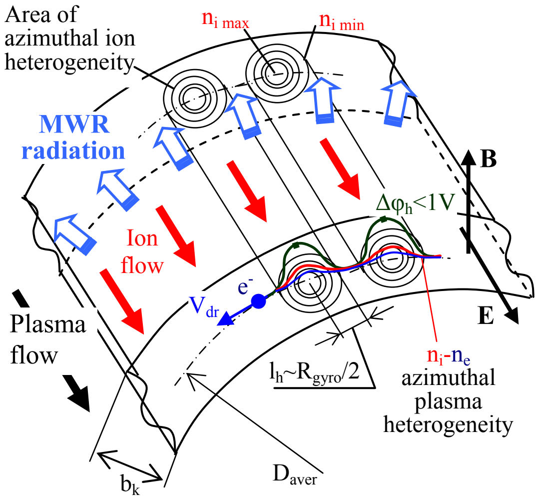


Figure 3. MWR radiation generated by electrons azimuthally drifted in crossed E×B fields and accelerated in local electric fields with azimuthal potential drop $\Delta\varphi_h$.

Full MWR power $W_h \approx 2 \cdot 10^{-11}$ from HT plasma is calculated as $W_h = w_h \cdot V_{pl} \cdot n_e \approx 2 \cdot 10^{-11}$ W, where characteristic value of electron concentration is $n_e \approx 5 \cdot 10^{17}$ m^{-3}. Thus, electromagnetic radiation of power $W_{hC} \approx 2 \cdot 10^{-11}$ W in frequency $\omega_h \geq 10 \cdot 10^9$ Hz, which is generated by separate electrons accelerated due to potential drop $\Delta\varphi_h \leq 1$ V in an azimuthal direction, is much less than that power registered in experiments 10^{-8} W in circular frequencies $\omega_{exp} \approx 9 \ldots 60$ GHz.

It is necessary to note that a local azimuthal electric field periodic influence (with change of a sign) to electrons, drifting in an azimuthal direction, can play a role of preliminary electron phasing. Such phasing, as consequence, can lead to the coherent mechanism of microwave radiation of electrons from each area of the extent $l_h \approx R_{le}/2$. Then power w_{hC} of coherent MWR of electrons, moving as a quasi-uniform charge of value $q = l_h^3 \cdot n_e \cdot e \approx 10^{-10}$ C with acceleration $a \approx \Delta\varphi_h / l_h \cdot e/m \approx 1.6 \cdot 10^{14}$ (m/s^2) through area l_h, is determined as $w_{hC} = (a \cdot q)^2 / (6 \cdot \pi \cdot \varepsilon_0 \cdot c^3) \approx 1.5 \cdot 10^{-7}$ W in dipole approach following methodic Ref. 8. Thus, full MWR power W_{hC} from HT plasma would be in limit $W_{hC} = w_{hC} \cdot N_{cell} \approx 10^{-1}$ W, where w_{hC} – power of coherent MWR from separate area, N_{cell} - quantity of separate radiating areas $N_{cell} = b_k / l_h \cdot L / l_h \cdot \pi \cdot D_{av} / l_h = V_{pl} / l_h^3 \approx 10^6$ in volume V_{pl} of plasma flow. It is necessary to notice what a problem about determining of possible preliminary phasing of electrons in local areas with an azimuthal electric field $\Delta\varphi_h / l_h$ is not so simple to solve it easy and quickly.

Such prospective "mechanism" of coherent radiation in frequency $\omega_h \geq 10 \cdot 10^9$ Hz of maximal power $W_{hC} \approx 10^{-1}$ W would almost overlap a range of experimentally registered MWR power – 1…0.1 W and $10^{-3 \ldots -8}$ W. It is necessary to notice that extent l_h of local areas varies in a range $(0.1 \ldots 1) \cdot R_{le} \approx 10^{-3 \ldots -4}$ m. Thus, possible range of MWR frequencies generated in plasma is $\omega_h \sim V_{edr} / (R_{le} \ldots R_{le}/10)$ that meets to experimentally registered MWR frequency range from ≈10 GHz to ≈120 GHz.

VII. Electromagnetic Radiation from HT Plasma – as Result of Mainly Longitudinal Waves Generated Because of Vavilov-Cherencov Effect (VCE)

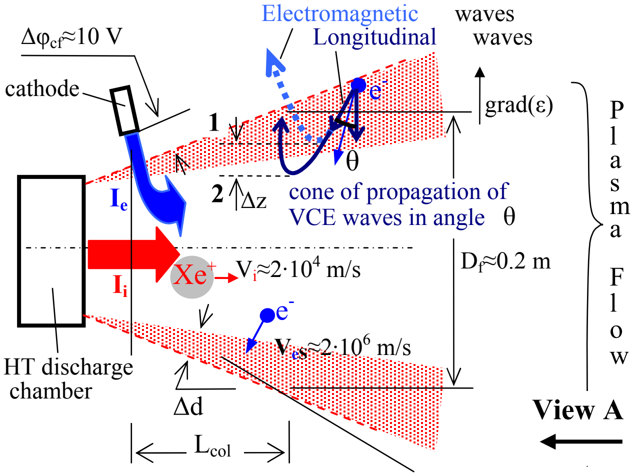
A. VCE Generation of Mainly Longitudinal Waves

Conditions of wave radiation and power of waves, generated owing to Vavilov-Cherenkov effect (VCE) are determined. These waves have the properties of mainly longitudinal (plasma) waves in plasma flow outside HT discharge chamber (further - medium). Efficiency of transformation of these waves into MWR is determined also. Following characteristic values of medium parameters and fields in analyzed area are used: electron temperature $T_e \approx 1$ eV, electric field $E \approx 10^3$ V/m, magnetic induction $B \approx 5 \ldots 1$ mTl, charge concentration $n_e \approx (10 \ldots 5) \cdot 10^{16}$ m^{-3}. By calculation it was supposed that the current $I_e \approx 2$ A of electrons emitted by the cathode, gets in flow with ion current $I_i \approx 2$ A outside the HT discharge chamber, collecting energy $\Delta\varphi_{pf} \approx 10$ V while passing through characteristic potential drop $\Delta\varphi_{cp} \approx 10$ V between the cathode and plasma flow. Thus, electron with energy $\approx \Delta\varphi_{cp}$, moving in the area of plasma where temperature is $T_e \approx 1$ eV, can generate longitudinal waves owing to VCE (electron velocity surpasses phase velocity of light in plasma) until electron energy will not decrease. Then these waves can be transformed in MWR which leaves area of plasma flow.

It is suppose that the external magnetic field is uniform and weak $\omega_B / \omega_{exp} << 1$, therefore plasma is considered isotropic and movement of electrons is rectilinear during length of a radiated wave. Relative difference of plasma concentration in a radial direction of wave generating area I shall estimate as $\Delta n_e / n_e \approx 1$. The condition of wave emission owing to VCE (abnormal effect) is determined, following methodic Ref. 7, as

$$n_{pf}\cdot\beta\cdot\cos(\theta) \geq 1,$$

where: $n_{pf}=(\varepsilon/(3\cdot\beta_T^2))^{1/2}=((1-(\omega_L/\omega)^2)/3)^{1/2}/\beta_T\approx 157$ - the refraction parameter of generated waves was determined as for plasma waves with phase velocity $V_{ph}=V_{eT}/(\varepsilon)^{1/2}=c/n_{pf}$ in medium they are generated (with dielectric permeability $\varepsilon\approx 0.29$ and Langmuir frequency $\omega_L=(n_e\cdot e^2/(\varepsilon_0\cdot m))^{1/2}\approx 12\cdot 10^9$ Hz), factor $\beta_T=V_{eT}/c$, in which $V_{eT}\approx(T_e\cdot e\cdot 2/m)^{1/2}\approx 6\cdot 10^5$ m/s - a thermal (chaotic) component of electron velocity in the medium; factor $\beta=V_{eS}/c\approx 6.7\cdot 10^{-3}$ in which $V_{eS}\approx(\Delta\varphi_{cp}\cdot e\cdot 2/m)^{1/2}\approx 2\cdot 10^6$ m/s - characteristic absolute electron velocity in a stream from the cathode when electron pass potential drop $\Delta\varphi_{cp}$ between the cathode and plasma flow; $\theta\approx 16^0$ - a critical angle between a wave emission direction and an electron movement direction (vector V_{eS}), by which VCE condition of wave emission (possessing properties of plasma waves) is satisfied.



Plasma parameters in area of VCE:
$B_r\approx 5\ldots 0$ mTl, $T_e\approx 1$ eV, $n_{e,i}\approx 5\cdot 10^{16}$ m^{-3},
Electron stream from cathode $I_e=I_i\approx 1\ldots 10$ A.

Figure 4. Radiation of electromagnetic waves caused by Vavilov-Cherenkov effect.

Frequency of emitted mainly longitudinal waves is determined, following methodic Ref. 7, as

$$\omega\approx V_{eS}\cdot k_{cp}\approx 15 \text{ GHz},$$

where: wave number $k_{pf}=n_{pf}\cdot\omega/c=7.5\cdot 10^3$ m^{-1} and $k_{pf}=2\cdot\pi/\lambda_{pf}$, $\lambda_{pf}=8.3\cdot 10^{-4}$ m - is length of wave in medium (plasma flow outside of discharge chamber).

Generated longitudinal waves will be underdamping as length of wave λ_{pf} considerably surpasses Debye radius of charge shielding (Ref. 7) $r_D\approx V_{eT}/\omega_L\approx 5\cdot 10^{-5}$ m $< \lambda_{pf}=8.3\cdot 10^{-4}$ m.

It is necessary to notice the following. Frequency ω of generated longitudinal waves will determine MWR frequency, which represent direct interest and which will arise by transformation longitudinal waves. VCE conditions - parameters of plasma flow and stream of electron from the cathode - assume disorder and also it is probable increasing of MWR frequency (relatively ω) by transformation of longitudinal waves. Because of this it is admissible to expect MWR frequency double increasing also. Thus, MWR frequency owing to VCE ≈30 GHz is close to extremely probable, whereas MWR of relatively low power in frequencies of ≥100 GHz was experimentally determined.

By determining of power W_{VCEL} of longitudinal waves, generated in plasma with maximum of radiated power in a range of frequencies $\Delta\omega\approx 3\cdot 10^9$ Hz close to ω (according to experimental results), it was considered the next. These longitudinal waves are generated owing to VCE by electrons with velocity $V_{eS}\approx(\Delta\varphi_{cp}\cdot e\cdot 2/m)^{1/2}$, emitted by the cathode as the stream $I_e/e\approx 1.3\cdot 10^{19}$ s^{-1}. It is suppose that: 1) VCE it is possible only due to electrons that still not losses its energy $\Delta\varphi_{cp}$ because of collisions with particles of plasma (in which electron transfers a part of its energy $\Delta\varphi_{cp}$ and loses ability to generate a wave); 2) losses on radiation are comparatively small. Thus, average time of electron radiation is determined by time τ_{col} of electron run until collision in plasma $\tau_{col}\approx 1/(n_e\cdot\sigma_{col}\cdot V_{eS})\approx 2.5\cdot 10^{-6}$ s, where $n_e\approx 2\cdot 10^{17}$ m^{-3} and cross-section of collisions is $\sigma_{col}\approx 10^{-18}$ m^2.

By V_{VCE} it was marketed volume of conditionally allocated plasma area close to plasma flow surface (look fig. 1, 2), in which VCE condition is satisfied and it is possible plasma wave generation by electrons during time τ_{col} since emission by cathode. Simplistically it was determine plasma area of MWR with temperature $T_e\approx 1$ eV, concentration $n_e\approx 5\cdot 10^{16}$ m^{-3} and volume V_{VCE} as a tubular flow of diameter $D_s\approx 0.2$ m thickness $\Delta d\approx 0.015$ m and length L_{col}, in which drifting in an azimuthal direction electrons keep its energy ($\Delta\varphi_{cp}$) necessary for VCE waves generation, so that $V_{VCE}\approx\pi\cdot D_s\cdot\Delta d\cdot L_{col}$. It was suppose that $V_{pl}\approx\pi\cdot D_s^2/4\cdot L_{col}$ - volume of plasma flow area containing completely trajectories of electron (emitted by the cathode and accelerated up to energy $\Delta\varphi_{cp}\approx 10$ eV) till the moment τ_{col} when electron loss of an essential share of energy $\Delta\varphi_{cp}$ so that it do not meet to VCE condition. Supposing that all electrons emitted by the cathode move with some average velocity V_{eS}, I have determined a share of time $d\tau_{col}$ of electron movements (when they are in the area of plasma in which VCE is possible) among the period τ_{col} (after which VCE is not possible) as $d\tau_{col}/\tau_{col}$. I estimate this share as $d\tau_{col}/\tau_{col}\approx V_{VCE}/V_{pl}\approx 4\cdot\Delta d/D_s\approx 0.33$. Thus, electrons are capable to VCE only during time $d\tau_{col}\approx 4\cdot\Delta d/D_s\cdot\tau_{col}$.

Power W_{VCEL} is determined following methodic Ref. 7, as

$$W_{VCEL} = \frac{e^2 \cdot \omega \cdot \Delta\omega}{V_{eS} \cdot \varepsilon \cdot 4 \cdot \pi \cdot \varepsilon_0} \cdot d\tau_{col} \cdot I_e / e \approx 0.15 \text{ (W)}$$

that, by the order of value, meets to power $W_{VCELc}=e^2 \cdot \omega_L{}^2/(8 \cdot \pi \cdot \varepsilon_0 \cdot V_{eS}) \cdot \ln(2/3 \cdot V_{eS}/V_{eT}$ radiated in all resolved range of frequencies (Ref. 7).

Let's estimate, following methodic Ref. 7, efficiency of Landau strengthening of power W_{VCEL} of longitudinal (plasma) waves generated owing to VCE by electron stream from the cathode in an flow of accelerated plasma outside HT discharge chamber (further – medium or plasma flow) by distribution of these waves thru medium. Instability of considered system "electron stream from the cathode – flow of accelerated plasma" (further - system "stream – plasma") is characterized by small indignation that is inserted by electron stream and will have convective character (carry away from area of generation and it will be absorbed not covering all area). The factor μ_{pf} (dimension of m^{-1}) of absorption/strengthening of plasma waves by charges system is determined by processes of "true" absorption and the induced emission. Increment $\mu_{pf}<0$ means strengthening of waves. The factor $\gamma=\mu_{pf} \cdot 3 \cdot V_{eT}{}^2/V_{ph}$ (dimension s^{-1}) characterizes absorption/increase of waves in time. Following methodic Ref. 7, μ_{pf} was calculate as

$$\mu_{pf} = \sqrt{\frac{\pi}{18}} \cdot \frac{\omega_{Ls}{}^2 \cdot 4 \cdot \pi \cdot \varepsilon_0 \cdot V_{ph}{}^3}{\omega \cdot V_{eT}{}^2 \cdot V_{eTs}{}^2} \cdot \left(V_{ph} - V_{es} \cdot \cos(\theta)\right) \cdot e^{\frac{-\left(V_{ph} - V_{es} \cdot \cos(\theta)\right)^2}{2 \cdot V_{eTs}{}^2}} \approx -1.4,$$

where: $\theta \approx 16^0$ - angle of a wave emissions cone by VCE; the index $_s$ – fall value into characteristics of electron stream from the cathode so that $\omega_{Ls}{}^2 \approx \omega_L{}^2/5=12^2/5 \cdot 10^9$ Hz, considering that concentration of charges in a stream of electron from the cathode (where ratio $n_e \approx n_i$ is determined by ion velocity $\approx 10^4$ m/s in this stream through cross-section $S \approx \pi \cdot D_f{}^2/4 \approx 3 \cdot 10^{-2}$ m^2) and in plasma flow from discharge chamber correspond as $\approx 1/(1\ldots2)$; approximately supposing a ratio of thermal electron velocities as $V_{eT}{}^2 \approx V_{eTs}{}^2$.

Strengthening of power W_{VCEL} of plasma waves will be determined by factor $\exp(-\mu_{pf} \cdot \Delta d)$, where $\Delta d \approx 0.015$ - characteristic thickness (radial) of a tubular plasma flow - areas of plasma in which it is going generation, distribution and strengthening of plasma waves. Thus, strengthened power of plasma waves was calculated as $W_{VCE} \approx W_{VCEL} \cdot \exp(-\mu_{pf} \cdot \Delta d) \approx 0.15 \cdot 1.1 \approx 0.17$ W.

B. Transformation of Longitudinal (plasma) Waves

Let's estimate, following methodic Ref. 7, efficiency Q_{TLSH} of transformation of power W_{VCEL} of plasma waves into radiated power $\mathbf{W_{RSH}}$ in the area of plasma flow with smooth change of plasma properties (**with a gradient of charge concentration in a radial direction of plasma flow**) outside HT discharge chamber so that $W_{RSH}=W_{VCEL} \cdot Q_{TLSH}$. Supposed that frequencies of generated microwaves and a plasma waves are close, efficiency Q_{TLSH} is determined as

$$Q_{TLSH} = \frac{1}{8} \cdot \left(\frac{V_{ph}}{c}\right)^2 \cdot \left(\frac{3 \cdot c \cdot \mathrm{grad}(\varepsilon)}{\omega}\right)^{2/3},$$

where: $V_{ph}=V_{eT}/(\varepsilon)^{1/2}=c/n_{pf}$ - phase velocity of plasma waves; ω - frequency radiated microwaves; $\varepsilon=1-(\omega_L/\omega)^2$; gradient of dielectric permeability between layers of plasma 1 and 2 longwise Δd, where there is a transformation of waves (look fig. 2), is $\mathrm{grad}(\varepsilon)=(\omega_{L1}{}^2-\omega_{L2}{}^2)/(\omega \cdot \Delta d)=(n_{e1}-n_{e2}) \cdot e^2/(\varepsilon_0 \cdot m \cdot \omega \cdot \Delta d)$. During period of an electrons movement $\tau_{col} \approx 2.5 \cdot 10^{-6}$ s in a plasma flow (look above) when it can initiate VCE, electrons move from the cathode with average velocity $V_i \approx 2 \cdot 10^4$ m/s on the distance $L_{col} \approx V_i \cdot \tau_{col} \approx 0.05$ m where there is characteristic value $(n_{e1}-n_{e2}) \approx (4\text{-}6) \cdot 10^{16}$. Thus, $|\mathrm{grad}(\varepsilon)| \approx 20$ m^{-1} and efficiency of transformation of plasma wave power into microwave radiation power will be $\mathbf{Q_{TLSH} \approx 7 \cdot 10^{-7}}$.

To check up a correctness of the estimation curried out above it is necessary to compare prospective extent Δz of wave transformation area to the cross-section size Δd of plasma wave generation area, where such transformation is possible. Following methodic Ref. 7, at $Q_{TLSH} \rightarrow \max$, extent $\Delta z \approx (2 \cdot c)^{2/3}/(3 \cdot \omega)^{2/3}/(\mathrm{grad}(\varepsilon))^{1/3} \approx 0.07$ m > $\Delta d \approx 0.015$ m that means that results of calculation do not fully meet to assumption accepted before calculation. However, this discrepancy can be corrected lowering factor $\Delta d/\Delta z$. The condition of sufficient "slowness" of plasma properties change is determined by an inequality $\omega/(c\cdot grad(\varepsilon))>>1$ that gives $15\cdot10^9/(3\cdot10^8\cdot20)=2.5>1$. It is expected that the demanded condition of "slowness" will be executed in longitudinal (to the accelerated plasma flow) direction of wave transformation area - the value of a gradient ε will be decrease as $grad(\varepsilon)/(10\ldots100)$ and efficiency of transformation will decrease - $Q_{TLSH}/(2\ldots5)$. Then, in view of correcting factors, power of microwaves in plasma I shall estimate as $\mathbf{W_{RSH}}=W_{VCEL}\cdot\mathbf{Q_{TLSH}}\cdot\Delta d/\Delta z/(2\ldots5)\approx0.15\cdot7\cdot10^{-7}\cdot0.015/0.07/3\approx\mathbf{10^{-8}}$ **(W).**

Let's estimate, following methodic Ref. 7, attenuation of MWR power $\mathbf{W_{RSH}}$ by passing through a stream of plasma in a radial direction up to an output of the plasma flow (in vacuum), i.e. attenuation during $D_f/2$ owing to collisions. Power W_{RV} radiated in vacuum is determined as $W_{RSHa}=W_{RSH}\cdot\exp(-\mu_\omega\cdot D_f/2)$, while characteristic decrement of power attenuation longwise unit of length is μ_ω. The value μ_ω is determined by electron collision frequency ν_m $\nu_m\approx V_{eT}\cdot\sigma_{col}\cdot n_e\approx1.2\cdot10^5$ s^{-1} (where $\sigma_{col}\approx2\cdot10^{-18}$ m^{-2} collision cross-section) as

$$\mu_\omega=\frac{e^2\cdot n_e\cdot\nu_m}{\varepsilon_0\cdot m\cdot c\cdot\left(\omega^2+\nu_m{}^2\right)}\approx10^{-3}\ (m^{-1}).$$

Decreasing of power, radiated in vacuum on border flow/vacuum, is determined by the factor $\exp(-\mu_\omega\cdot D_f/2)\approx\exp(-10^{-3}\cdot0.1)\approx1$ (i.e. waves practically are not weakened through the plasma flow up to an output in vacuum) and power is $W_{RSHa}=W_{RSH}\cdot\exp(-\mu_\omega\cdot D_f/2)\approx10^{-8}$ W.

Let's estimate, following methodic Ref. 7, efficiency Q_{TR} of transformation of plasma wave power W_{VCEL} into MWR power W_{RR} ($W_{RR}=W_{VCEL}\cdot Q_{TR}$) **owing to Rayleigh scatering of plasma waves on fluctuations of ion concentration** in the area of volume $V=l_f^3$ with the linear size $l_f\approx10^{-4}$ m smaller than plasma wave $\lambda_{pf}=8.3\cdot10^{-4}$ m and greater than Debye radius $r_D\approx V_{eT}/\omega_L\approx6\cdot10^{-5}$ m in plasma flow outside HT discharge chamber. While $l_f<<\lambda_{pf}$, electrons in volume of plasma V will radiate microwaves as a uniform dipole and fluctuation of ions in this volume is stochastic slow process (comparison to frequency ω_L). Then, efficiency of transformation Q_{TR} is determined by expression

$$Q_{TR}=\frac{e^4\cdot n_i\cdot l_f}{3\cdot m^2\cdot c^3\cdot V_{eT}\cdot4\cdot\pi\cdot\varepsilon_0{}^2}\approx\mathbf{10^{-13}},$$

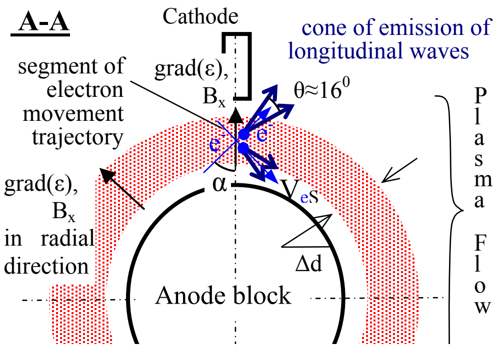


Figure 5. Characteristic direction of plasma wave propagation in area of its generation (look also fig. 4).

where: $V_{eT}\approx6\cdot10^5$ m/s - a thermal (chaotic) component of electron velocity, $n_i\approx n_e\approx5\cdot10^{16}$ m^{-3}. Then, power of microwave radiated in vacuum from plasma is $W_{RR}=W_{VCEL}\cdot Q_{TR}\approx10^{-14}$ W. By this transformation the frequency of generated MWR wave and a plasma wave is admissible to not distinguish.

Let's estimate, following methodic Ref. 7, **efficiency $\mathbf{Q_{TLSHM}}$ of transformation of plasma wave** power W_{VCEL} into MWR power W_{RM} **in view of magnetoactive plasma with smooth change of concentration heterogeneity** (similarly to made above) outside HT discharge chamber, while frequencies of generated MWR wave and a plasma wave are close. It was suppose that the magnetic field distribution with characteristic induction $B_x\approx1.5$ mTl of an external source is uniform and weak (as $\omega_B/\omega\approx0.015<<1$ and a plasma wave length $\lambda_{cp}=8.3\cdot10^{-4}$ m is much less than gyroradius $R_{le}\approx10^{-2}$ m). It was suppose also that distribution of plasma waves (along a magnetic field) is going along a gradient of concentration. The parameter $2\cdot\delta_{01}$ of interactions falling plasma wave and scattering electromagnetic wave is determined by: gyrofrequency $\omega_B=e\cdot B_x/m\approx0.25$ GHz (when characteristic induction $B_x\approx1.5$ mTl); gradient of dielectric permeability that was taken as a first approximation $grad(\varepsilon)\cdot\cos(\alpha\approx45^0)\approx20/1.4\approx14$ m^{-1} in a direction of wave propagation; α - angle between direction B_x and a direction of plasma wave propagation at small θ, supposing $tg(\alpha)\approx\alpha$ (look fig. 3), as

$$2\cdot\delta_{01}\approx\frac{\pi}{2}\cdot\frac{\omega}{c\cdot grad(\varepsilon)}\cdot\frac{\alpha^2}{\left(1+\omega/\omega_B\right)^{3/2}}\approx\alpha^2\cdot10^{-2}.$$

The factor of transformation $\mathbf{Q_{TLSHM}}(\alpha)$ was determined as

$$Q_{TLSHM}(\alpha) = e^{-2\cdot\delta_{01}} \cdot \left(1 - e^{-2\cdot\delta_{01}}\right).$$

Average value $Q_{\mathbf{TLSHM}}$ was determined by α in a range from 0 up to/2 (in view of that in the field of interaction as the magnetic field has a various orientation, and plasma waves have a wide angular spectrum of distribution) and believing parameter$2\cdot\delta_{01}$ small value,

$$Q_{TLSHM} \approx \int_0^{\pi/2} Q_{TLSHM}(\alpha)\cdot d\alpha \bigg/ \int_0^{\pi/2} d\alpha \approx \int_0^{\pi/2} \left(1-\alpha^2\cdot 10^{-2}\right)\cdot\alpha^2\cdot 10^{-2}\cdot d\alpha \bigg/ (\pi/2) \approx 10^{-2}\cdot\pi^2/12 \approx 7.5\cdot 10^{-3}.$$

In view of return reflection I have estimated microwave power radiated from HT plasma as $\mathbf{W_{RSHMr}}=W_{RSHM}\cdot\exp(-\mu_\omega\cdot D_\pi/2)=W_{VCEL}\cdot Q_{TLSHM}\cdot\exp(-10^{-3}\cdot 0.1)\approx 10^{-3}$ (W).

Thus, the microwave radiation from HT plasma registered experimentally in a range of circular frequencies $\omega_{exp}\geq 12$ GHz and power $10^{-8\ldots-3}$ W can be explain by generation of mainly longitudinal waves owing to Vavilov-Cherencov effect and their subsequent transformation into microwaves in the same frequency in the area of smooth heterogeneity of magnetoactive plasma flow. Frequency and power of generated MWR are determined by features of the interconnected HT units operation – cathode and anode: current I_e of electrons from the cathode, potential drop $\Delta\varphi_{cp}$ between the cathode and accelerated plasma flow, temperature T_e of electrons in accelerated plasma flow, volume and thickness of a plasma-vacuum boundary layer in accelerated plasma flow, radial component of magnetic field induction B_r in this boundary layer, charge concentration varying that determine gyrofrequency $\omega\geq 12$ GHz similar to the experimental range ω_{exp}.

VIII. Conclusion

The most probable reasons of electromagnetic microwave radiation from HT plasma (in a range of frequency $f\approx 1.5\ldots 10$ GHz and circular frequency $\omega_{exp}\approx 9\ldots 60$ GHz, of power $W_{exp}\approx 10^{-8\ldots-3}$ W or 0.1…1 W that was registered experimentally), which can be generated directly owing to acceleration of electrons are next: 1) acceleration of electrons in a boundary layer of plasma in HT discharge chamber that can cause radiation $W_{BL}\approx 10^{-8}$ W; 2) possible quasi–cophased electron acceleration in an azimuthal direction in local areas (extent less than electron Larmor radius and with potential drop in an azimuthal direction ≤ 1 V) that can cause coherent radiation from such area of the maximal total radiation power $W_{hC}\leq 0.1$ W in a range of frequencies up to $\omega\leq 120$ GHz whereas peak spectral density of radiation power is about $\omega_h\approx 12\cdot 10^9$ Hz.

Transformation of mainly longitudinal (plasma) waves, generated in HT plasma flow, into microwave have been studied also. Such transformation in a layer of smoothly-heterogeneities magnetoactive plasmas (outside HT discharge chamber close to the cathode) can lead to the microwave radiation from HT plasma of power $W_{VCER}\leq 10^{-3}$ W in frequencies $\omega\leq 30$ GHz. Mainly longitudinal (or plasma waves) can be generated in the same layer owing to Vavilov-Cherenkov effect by electrons emitted by the cathode into HT accelerated plasma flow.

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