

Ion Beam Instability in Hall Thrusters

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Abstract: Stability of an ion flux, bounded with an anode and cathode, in Hall thrusters is investigated by theoretical and numerical modeling. Two-fluid MHD approximation with cold magnetized electrons and cold non-magnetized ions is used. The inertia of the electrons is taken into consideration. The perturbations are assumed to be quasineutral, potential, and dependent on a single spatial coordinate only. For simplicity, the magnetic field is assumed to be uniform. It is shown that the presence of the boundaries, where the potential of the ion beam is fixed, can cause instability of the beam. The growth rate of instability and excited oscillation frequency is of the order of the reciprocal of the time required for the ion to pass the distance between the anode and cathode. In the limit of a uniform state of the unperturbed ion flux, the instability is analogous to the Pierce instability of an electron beam with the fundamental distinction that here, the perturbations are quasineutral. It is shown that the instability is alternately “aperiodical” and “oscillating” depending on the range of the α_{LH} parameter, which includes induction of the magnetic field (through lower hybrid frequency), thickness of the acceleration layer, and velocity of the ion flux. The aperiodic instability is expected to occur in modern Hall thrusters. It should bring about change in distribution of the potential and plasma parameters in the acceleration layer. The oscillating instability is more inherent to the extended acceleration layer, typical for the models of the first generation. In its properties, it corresponds to the instability, which drives the so-called transit-time oscillations that dominated the spectrum of the oscillations in the Hall thrusters of the first generation.

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I. Introduction

NOWADAYS, Electric propulsion is successfully used in space missions and the scale of its applications grows steadily. Among the Electric propulsions, the Hall thruster is one of the most sought after. It possesses high efficiency and acceptable lifetime.

The further improvement of efficiency of the Hall thruster requires knowledge of physical processes in plasma of the thruster in fine detail. In the Hall thrusters, the plasma is far from equilibrium and is placed in a rather strong magnetic field. Both features create conditions for arising plasma instabilities. The instabilities, in turn, influence the performance of the thruster and its compatibility with the Electric propulsion subsystems and the electronic equipment of the spacecraft. Therefore, the instabilities in the Hall thrusters were and are the topic of a large body of research. The results of some of these studies are described in the review by Choueiri¹. Nevertheless, in spite of more than four decades of instabilities investigations in Hall thrusters, their physics is still not fully understood. One of the main reasons for this, in our opinion, is that many theoretical models of the instabilities ignore the fact that real unstable perturbations in most cases are large scale and as a consequence need to take into account boundary conditions for the perturbations. The influence of the simplest boundary conditions for one-dimensional perturbations on the stability of the ion flux in the Hall thruster is the topic of the present paper.

II. Theoretical Model of Ion Beam Instability

At building a theoretical model, we use the two-fluid MHD approximation with cold magnetized electrons and cold non-magnetized ions. We restrict ourselves to quasineutral, potential perturbations and neglect dissipative and ionization processes. The perturbations are assumed to be dependent on a single spatial coordinate. We use Cartesian coordinates with the X and Z axes directed along the directions of the unperturbed ion velocity and the applied magnetic field, respectively. For simplicity, the unperturbed velocity V_0 , density n_0 of the ions, and magnetic field are assumed to be uniform. Under these assumptions, linearized MHD equations take the following form:

$$\frac{\partial V_x}{\partial t} + V_0 \frac{\partial V_x}{\partial x} = -\frac{e}{M} \frac{\partial \Phi}{\partial x} \quad (\text{II.1})$$

$$\frac{\partial n}{\partial t} + n_0 \frac{\partial V_x}{\partial x} + V_0 \frac{\partial n}{\partial x} = 0 \quad (\text{II.2})$$

$$\frac{\partial u_x}{\partial t} = \frac{e}{m} \frac{\partial \Phi}{\partial x} - \frac{e}{m} u_y B_0 \quad (\text{II.3})$$

$$\frac{\partial u_y}{\partial t} = \frac{e}{m} u_x B_0 \quad (\text{II.4})$$

$$\frac{\partial n}{\partial t} + n_0 \frac{\partial u_x}{\partial x} = 0 \quad (\text{II.5})$$

Where Φ – the perturbation of potential,
 V_x – the projection of ion velocity on axis X ,
 n – the number density of ions (electrons),
 e – the unit positive charge,

u_x, u_y – the projections of electron drift velocity on axes X and Y respectively,
 m – the electron mass,
 B_0 – the induction of magnetic field.

In the Eqs. (II.1) – (II.5), the quantities without index “0” are the perturbations of corresponding parameters, while those with index “0” – non-perturbed parameters.

The boundary conditions to the set of the equations (II.1) – (II.5) can be written as follow:

$$\Phi(0) = \Phi(L) = 0; \quad n(0) = 0; \quad V_x(0) = 0, \quad (\text{II.6})$$

where L – the distance between the anode and cathode.

We seek the solution of the set of Eqs. (II.1) – (II.5) in the form:

$$\mathbf{F}(x, t) = \mathbf{F}_j(x)e^{-i\omega t} \quad (\text{II.7})$$

Where \mathbf{F} is the vector of perturbed parameters, and vector \mathbf{F}_j is its x part only.

$$\text{We assume also that } \left| \frac{\omega}{\omega_{Be}} \right| \ll 1, \quad (\text{II.8})$$

where $\omega_{Be} = \frac{eB_0}{m}$ – the electron cyclotron frequency.

This assumption enables one to ignore the square of frequencies ratio in comparison with 1. As a result, we obtain after some transformations instead of the set (II.1) – (II.5) the following set of equations:

$$-i\omega V_x + V_0 \frac{\partial V_x}{\partial x} = -\frac{e}{M} \frac{\partial \Phi}{\partial x}, \quad (\text{II.9})$$

$$-i\omega n + n_0 \frac{\partial V_x}{\partial x} + V_0 \frac{\partial n}{\partial x} = 0, \quad (\text{II.10})$$

$$\frac{\partial^2 \Phi}{\partial x^2} = -\frac{m\omega_{Be}^2}{en_0} n \quad (\text{II.11})$$

The set (II.9) – (II.11) is a set of the ordinary differential equations with constant coefficients. At first, we assume that a wave length λ of the perturbation is small enough, such that one can neglect λ in comparison with L . This means that we can neglect the effect of boundaries and use $\mathbf{F}_j(x) = \mathbf{F}_j e^{ikx}$ in Eqs (II.9)-(II.11).

Where \mathbf{F}_j – the vector of Fourier component of perturbed parameters, $k = \frac{2\pi}{\lambda}$.

As a result, we obtain the following dispersion equation:

$$(\omega - kV_0)^2 = \omega_{Be}\omega_{Bi} \quad (\text{II.12})$$

$$\omega_{1'} = \sqrt{\omega_{Be}\omega_{Bi}} + kV_0$$

$$\omega_{2'} = -\sqrt{\omega_{Be}\omega_{Bi}} + kV_0$$

Where $\omega_{Bi} = \frac{eB_0}{M}$.

From the Eq. (II.12), it is evident that for short wave perturbations and under the above assumptions, the ion flux in the Hall thruster is stable ($Im \omega = 0$).

Now, we pass on to the case of $\lambda \sim L$. We need to solve the boundary eigenvalue problem (II.9) - (II.11) with (II.6). It can be done in the following manner. The general solution of the set (II.9)-(II.11) is found in the ordinary way. Then, using the boundary conditions (II.6), we obtain a set of algebraic equations for determination of arbitrary constants. Equating to zero a determinant of this set, one can derive a dispersion equation.

The set (II.9) – (II.11) together with the boundary conditions (II.6) corresponds formally to the set of equations and boundary conditions describing the so-called Pierce instability² (See also the book of Mikhailovskii³) of the compensated electron beam, moving in a gap between a cathode and anode but with three distinctions:

- 1) in the equations of motion and continuity instead of the charge, mass, and velocity of the electron, a charge, mass, and velocity of the ion appeared;
- 2) in the equation (II.11), the coefficient in the second term is $\frac{m\omega_{Be}^2}{en_0}$, not $\frac{e}{\varepsilon_0}$;
- 3) not only the unperturbed state, but also the perturbations of the plasma are quasineutral.

The Pierce instability is well studied. Taking into account the noted above analogy between the instability of the ion flux in the Hall thrusters and the classical Pierce instability, we can write a dispersion equation for the instability arising in the plasma of the thruster without performing rather cumbersome transformations. We start from the dispersion equation of the classical Pierce instability. It takes the following form³:

$$2\xi\alpha(1 - e^{i\xi} \cos \alpha) + ie^{i\xi}(\xi^2 + \alpha^2)\sin \alpha + i\frac{\xi^2}{\alpha}(\xi^2 - \alpha^2) = 0 \quad (\text{II.13})$$

Where
$$\xi = \frac{L\omega}{V_{e0}} \quad \alpha = \frac{L\omega_{pe}}{V_{e0}} \quad (\text{II.14})$$

$$\omega_{pe} = \sqrt{\frac{n_0 e^2}{\varepsilon_0 m}} \text{ - the electron Langmuir frequency,}$$

ε_0 - the dielectric constant of vacuum.

Pierce showed that if α meets the condition

$$(2N - 1)\pi < \alpha < 2N\pi, \quad (\text{II.15})$$

where $N = 1, 2, 3, \dots$;

from the dispersion equation, an aperiodic instability of the electron beam follows with a maximum growth rate of $\gamma \approx \frac{V_{e0}}{L}$.

Later, Pierce's results were supplemented by different authors (See a review in the book of Nezhlin⁴). It was found that the dispersion equation admits also an oscillatory instability. Those ranges of variations of α in which the oscillatory instability dominates are complementary with respect to the aperiodic instability. They meet the condition

$$2N\pi < \alpha < (2N + 1)\pi \quad (\text{II.16})$$

$N = 1, 2, 3, \dots$

In our case of the ion beam instability instead of the dispersion equation (II.13) and conditions (II.15) and (II.16) we should write

$$2\xi_{ion}\alpha_{lh}(1-e^{i\xi_{ion}}\cos\alpha_{lh})+ie^{i\xi_{ion}}(\xi_{ion}^2+\alpha_{lh}^2)\sin\alpha_{lh}+i\frac{\xi_{ion}^2}{\alpha_{lh}}(\xi_{ion}^2-\alpha_{lh}^2)=0 \quad (II.17)$$

$$(2N-1)\pi < \alpha_{lh} < 2N\pi \text{ - the condition for aperiodic instability} \quad (II.18)$$

$$2N\pi < \alpha_{lh} < (2N+1)\pi \text{ -the condition for oscillatory instability domination,} \quad (II.19)$$

$$N = 1, 2, 3, \dots$$

$$\text{where } \alpha_{lh} = \frac{\omega_{lh}}{V_0}L, \quad \omega_{lh} = \sqrt{\omega_{Be}\omega_{Bi}} \text{ - lower hybrid frequency, } \xi_{ion} = \frac{L\omega}{V_0}.$$

The growth rate of the instability is $\gamma \approx \frac{V_0}{L}$.

Since in our case, as distinct from the Pierce instability, the perturbations of the plasma are quasineutral, a principal question appears: how in the one-dimensional, collisionless problem the approximate compensation of the ion perturbations by electrons is provided in spite of a magnetic field, transverse to the direction of ion motion? The answer is as follows. As the electrical field in the oscillation or at the aperiodic growth of the perturbation changes, the velocity of the electrical drift of the electrons in the azimuthal direction changes as well. As a consequence of the finite mass of the electrons, the inertia force appears. The force causes the so-called inertia drift of the electrons. The drift is directed transverse to both the magnetic field and the direction of the inertia force, that is, in the X - direction. This drift provides the quasineutrality of the perturbations.

The features of the instability will be considered in more detail in Sec. III.

III. Numerical Modeling of Ion Beam Instability

The results, obtained in the previous section, concern solving the simplest problem: the ion beam propagates with the constant velocity V_0 across the magnetic field, between the anode and cathode. Here, we consider a more realistic problem. We research the stability of the ion flux where the ions in the unperturbed state are accelerated by the constant electrical field, applied between the anode and cathode of the Hall thruster. As before, the magnetic field is assumed to be uniform. Then, instead of the linearized MHD equations for the ion component (II.1-II.2), we should write

$$\frac{\partial V_x}{\partial t} + V_0 \frac{\partial V_x}{\partial x} + V_x \frac{\partial V_0}{\partial x} = -\frac{e}{M} \frac{\partial \Phi}{\partial x} \quad (III.1)$$

$$\frac{\partial n}{\partial t} + n_0 \frac{\partial V_x}{\partial x} + V_0 \frac{\partial n}{\partial x} + V_x \frac{\partial n_0}{\partial x} + n \frac{\partial V_0}{\partial x} = 0, \quad (III.2)$$

$$\text{Here, } V_0 = \sqrt{V_{00}^2 + \frac{2eE_0x}{M}} \quad (III.3)$$

$$n_0 = \frac{n_{00}V_{00}}{V_0} \quad (III.4)$$

V_{00} – the unperturbed velocity of ions at $x=0$,

n_{00} – the unperturbed density of ions at $x=0$.

As a consequence of the non-uniformity of n_0 , the continuity equation of the electron flux (II.5) should be changed to:

$$\frac{\partial n}{\partial t} + n_0 \frac{\partial u_x}{\partial x} + u_x \frac{\partial n_0}{\partial x} = 0. \quad (III.5)$$

Solving the set of equations (III.1)-(III.2), (II.3)-(II.4), (III.5), while taking into account Eqs (III.3)-(III.4) and inequality (II.8), we seek the solution as before in the form of (II.7). As a result, after transformations and introducing dimensionless quantities, we obtain a new set of equations:

$$\frac{\partial V_1}{\partial x_1} = -\frac{1}{qx_1 - q + 1} \frac{\partial \Phi_1}{\partial x_1} + i\omega_1 \frac{1}{(qx_1 - q + 1)^{1/2}} V_1 - \frac{q}{qx_1 - q + 1} V_1 \quad (\text{III.6})$$

$$\frac{\partial n_1}{\partial x} = \frac{1}{qx_1 - q + 1} \frac{\partial \Phi_1}{\partial x_1} - i\omega_1 \frac{1}{(qx_1 - q + 1)^{1/2}} V_1 + i\omega_1 \frac{n_1}{(qx_1 - q + 1)^{1/2}} + \frac{q}{qx_1 - q + 1} V_1 \quad (\text{III.7})$$

$$\frac{\partial^2 \Phi_1}{\partial x_1^2} - \frac{q}{2(qx_1 - q + 1)} \frac{\partial \Phi_1}{\partial x_1} + \alpha_{LH}^2 n_1 = 0 \quad (\text{III.8})$$

The boundary conditions (II.6) transform to the following

$$\Phi_1(0) = \Phi_1(L) = 0; \quad n_1(0) = 0; \quad V_1(0) = 0, \quad (\text{III.9})$$

$$\text{where } x_1 = \frac{x}{L}, \quad t_1 = \frac{tV_{0L}}{L}, \quad V_1 = \frac{V_x}{V_{0L}}, \quad \Phi_1 = \frac{\Phi e}{MV_{0L}^2}, \quad \omega_1 = \omega \frac{L}{V_{0L}},$$

$$\omega_1 = \omega_{r1} + i\gamma_1$$

$$V_{0L} = \sqrt{\frac{2eE_0L}{M} + V_{00}^2} \quad \text{- the unperturbed velocity of ions at } x = L,$$

$$q = \frac{2eE_0L}{MV_{0L}^2} \quad \text{- the fraction of energy of unperturbed ions, due to acceleration}$$

in unperturbed electrical field, in the total energy of unperturbed ions at $x = L$.

(Hereafter, we can consider L as thickness of an acceleration layer.)

The boundary eigenvalue problem (III.6-III.9) is solved numerically using the shooting method combined with so-called global-converging Newton-Ralphson iteration procedure.

Calculations were carried out for a sufficiently large region of α_{LH} values (from 0.7π to 3π) and for two values of q : $q = 0$ and $q = 0.96$. It is necessary to emphasize that for both values of q the energy of the unperturbed ions at $x = L$ was the same.

Numerical calculations for $q = 0$, for which an analytical solution is available, were carried out to compare the results for $q = 0$ and $q = 0.96$ at the same accuracy of calculations.

Table 1.

α_{LH} / π	$\omega_{r1} (q=0)$	$\gamma_1 (q=0)$	$\omega_{r1} (q=0.96)$	$\gamma_1 (q=0.96)$
0.715	-	-	0	0
0.725	-	-	0	0.017
0.750	-	-	0	0.059
0.800	-	-	0	0.139
0.850	-	-	0	0.206
0.900	-	-	0	0.259
0.950	-	-	0	0.299
1.000	0	0	0	0.325
1.050	0	0.120	0	0.336
1.100	0	0.224	0	0.332

1.150	0	0.325	0	0.306
1.200	0	0.414	0	0.253
1.250	0	0.495	0	0.148
1.275	0	0.532	0	0.048
1.290	0	0.553	-	-
1.300	0	0.566	-	-
1.350	0	0.620	-	-
1.400	0	0.683	0.549	0.025
1.450	0	0.729	0.626	0.068
1.500	0	0.765	0.688	0.094
1.550	0	0.792	0.732	0.107
1.600	0	0.810	0.767	0.101
1.650	0	0.817	0.783	0.067
1.700	0	0.812	0.798	0.003
1.750	0	0.790	-	-
1.800	0	0.758	-	-
1.850	0	0.705	0	0
1.900	0	0.616	0	0.146
1.950	0	0.473	0	0.262
2.000	0	0	0	0.345
2.050	0.571	0.095	0	0.405
2.100	0.766	0.201	0	0.443
2.150	0.926	0.292	0	0.466
2.200	1.037	0.356	0	0.470
2.250	1.131	0.426	0	0.462
2.300	1.208	0.472	0	0.429
2.350	1.278	0.518	0	0.368
2.400	1.338	0.550	0	0.255
2.450	1.385	0.575	0	0
2.500	1.430	0.591	0.485	0
2.550	1.466	0.591	0.630	0.087
2.600	1.498	0.579	0.717	0.165
2.650	1.526	0.553	0.775	0.216
2.700	1.551	0.508	0.816	0.244
2.750	1.572	0.442	0.842	0.252
2.800	1.592	0.344	0.857	0.238
2.850	1.630	0.198	0.861	0.199
2.900	1.713	0	0.851	0.119
2.950	-	-	0.821	0

The results of numerical modeling are presented in Table 1 and in Fig.1 – Fig.5. For the plots in Fig.1 - Fig.5, the free parameters $\text{Re} \frac{\partial \Phi_1}{\partial x_1}(0)$ and $\text{Im} \frac{\partial \Phi_1}{\partial x_1}(0)$ were chosen with the opposite signs to provide a better presentation of the results.

In Table 1, the frequencies and growth rates of the instabilities are given versus α_{ih}/π . If the growth rate is less than zero in a given region, it is denoted with a sign “-“. From the data, presented in Table 1, it follows that in the real case of an acceleration of the unperturbed ions (similar to the case of the ions

moving with constant velocity) the instability is alternately “aperiodical” and “oscillating” depending on the range of α_{LH}/π . At $q = 0.96$, the period of change between instability types is less than at $q = 0$. The threshold of the instability at $q = 0.96$ is lower as well. Both these features can be explained by the fact that at the same unperturbed velocity at $x = L$, the average velocity of the ions in the acceleration layer at $q = 0.96$ is less than at $V_0 = \text{const}$. The growth rates in the case of $q = 0.96$ are also less than they are at $q = 0$. Moreover, reducing the growth rate is stronger than reducing the average velocity. The latter is probably due to a degradation of a resonance condition at the acceleration of the ions. Nevertheless, even at $q = 0.96$, the growth rates of both aperiodical and oscillating instabilities remain quite high. At maximum, both growth rate and frequency of the excited oscillation are of the order of the reciprocal of the time needed for the ions to cross the acceleration layer.

In Fig.1 – Fig.4, the spatial structures of the unstable perturbations (real and imaginary parts of Φ_1) at $q = 0.96$ are shown for the different values of α_{LH}/π . It can be seen that as α_{LH}/π increases the spatial profile of the perturbation becomes more complex, namely the number of spatial oscillations increases. A comparison of two perturbation profiles related to the same type of the instability but with $q = 0.96$ and $q = 0$ (at the nearest ranges of α_{LH}/π) demonstrates the surprising similarity in appearance. Especially, this holds for the aperiodical instability (See Fig.5).

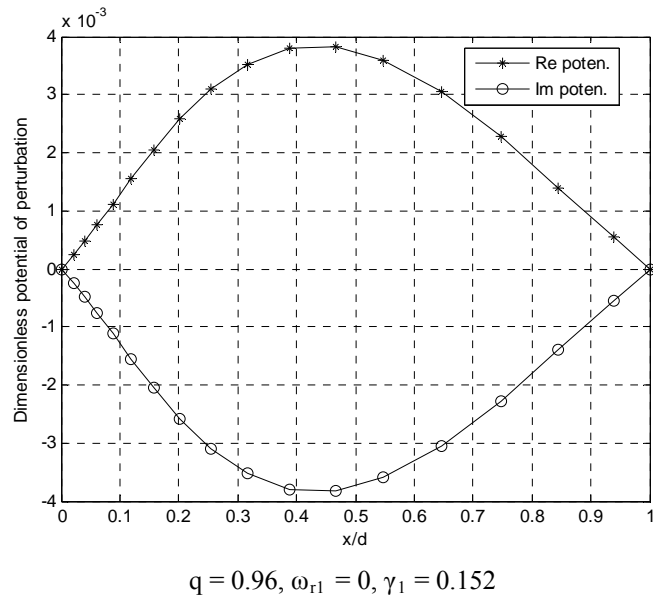


Fig.1. Spatial profile of perturbation ($\alpha_{LH} = 0.808$)

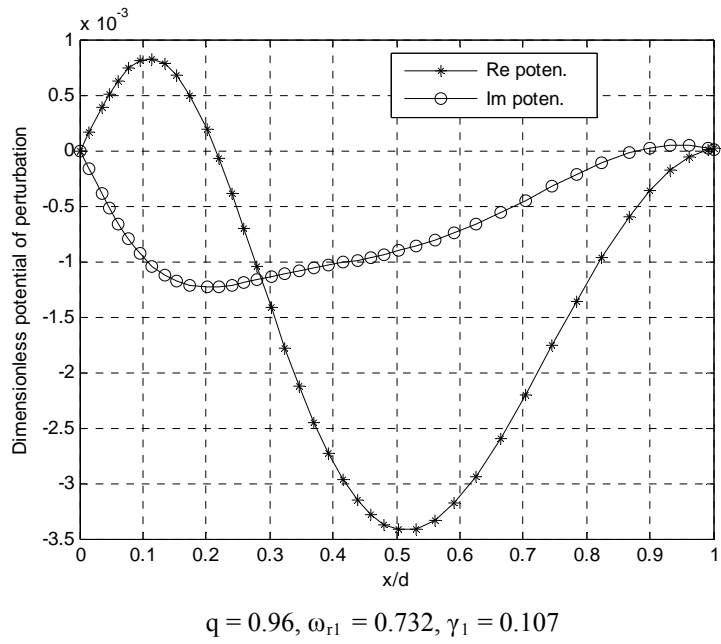


Fig.2. Spatial profile of perturbation ($\alpha_{LH} = 1.55$)

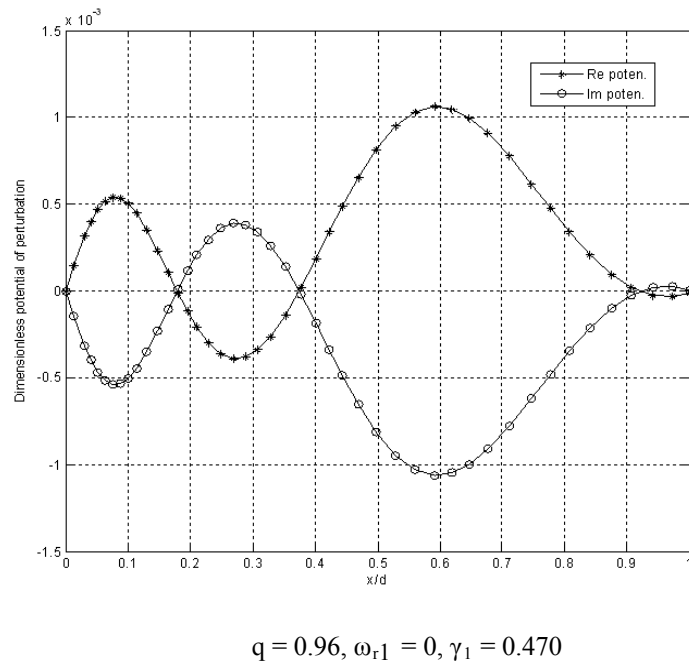


Fig.3. Spatial profile of perturbation ($\alpha_{LH} = 2.2$)

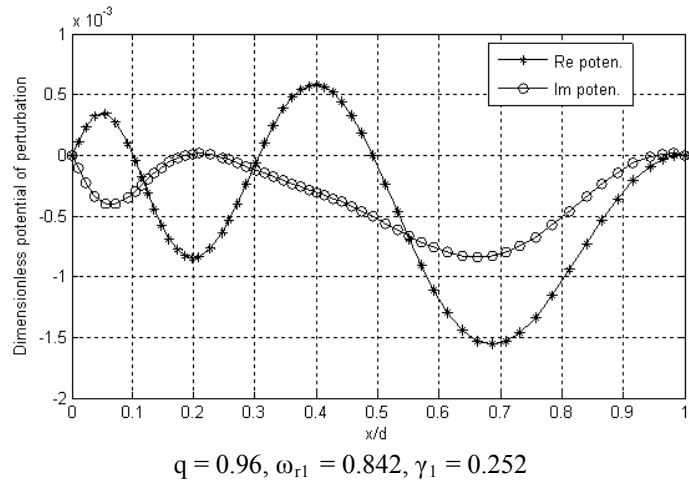


Fig.4. Spatial profile of perturbation ($\alpha_{LH} = 2.75$)

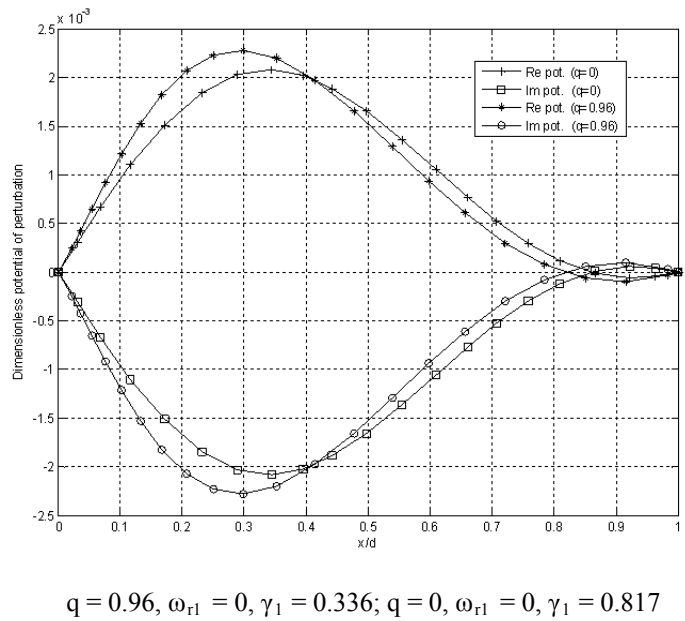


Fig.5. Spatial profile of perturbation ($\alpha_{LH} = 1.05, q = 0.96, \alpha_{LH} = 1.65, q = 0$)

To be more specific in the analysis of the results, we will consider two numerical samples:

1. In the modern Hall thrusters, the optimal thickness of the acceleration layer is $\sim 10^{-2}$ m for not too small diameters of the channel⁵. Then at $B_0 = 0.016$ T and $V_{0L} = 1.85 \cdot 10^4$ m/s, we have $\alpha_{LH}/\pi = 0.808$. In accordance with Table 1, this α_{LH}/π value corresponds to the aperiodic instability with $\gamma_1(q = 0.96) = 0.152$. The aperiodic instability should bring about a redistribution of the electric potential and the parameters of the plasma in the acceleration layer, which can be determined by solving a non-linear problem. At the distribution of the perturbed potential, which is shown in Fig.1, the initially uniform electrical field should transform to the field with the lower value at the beginning of the acceleration layer and higher value at the end of the layer. Thus, even in the uniform magnetic field (!), the instability of the ion beam can lead to a localization of the electric field predominantly near the exit of the acceleration channel.
2. In the Hall thrusters of the first generation⁶, the thickness of the acceleration layer essentially exceeded that in the modern thrusters. This was due to a larger extension of the magnetic poles. Besides, the velocity of (Xe) ions was lower. Assuming $L = 1.6 \cdot 10^{-2}$ m, $V_{0L} = 1.54 \cdot 10^4$ m/s, and $B_0 = 0.016$ T, we have $\alpha_{LH}/\pi(q = 0.96) = 1.55$. At this value of α_{LH}/π , the instability is oscillating with the following parameters $\gamma_1 = 0.107$, $\omega_{r1} = 0.732$ or $f = \frac{\omega_{r1} V_{0L}}{2\pi L} = 1.12 \cdot 10^5$ Hz. The properties of oscillations following from the above given theoretical model of the instability: frequencies, threshold of exciting (including the fact that the oscillations are existed in the thrusters with the extended magnetic poles and are absent in the thrusters with the narrow poles), the absence of time shifts over the entire acceleration layer (standing waves, see Fig.2) correspond to the features of the so-called transit-time perturbations⁷, which dominated the spectrum of oscillations in the first generation Hall thrusters. The models, suggested previously, for the instability, which is responsible for exciting the transit-time oscillations, are not convincing enough, because from all properties listed above, they can explain the frequencies of the oscillations only.

IV. Conclusions

As a result of the presently conducted investigations:

1. A theoretical model of ion beam instability in Hall thrusters was developed. The cause of the instability is influence of boundaries with fixed electrical potentials on an ion flux.
2. It was shown that in the limit of a uniform state of the unperturbed ion flux, the instability is analogous to the Pierce instability of an electron beam with the fundamental distinction that here, perturbations are quasineutral.
3. It was shown that the instability is alternately “aperiodical” and “oscillating” depending on the range of the α_{LH} parameter, which includes induction of the magnetic field (through a lower hybrid frequency), thickness of the acceleration layer, and velocity of the ions.
4. The aperiodic instability should occur in the modern Hall thrusters. It should bring about a change in the distribution of the potential and the parameters of the plasma in the acceleration layer.
5. The oscillating instability is more inherent to models with the extended acceleration layer, typical for the Hall thrusters of the first generation. In its properties, it corresponds to the instability, which drives the so-called transit-time oscillations, which dominated the spectrum of the oscillations in the Hall thrusters of the first generation.

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